OPTIMAL DESIGN OF VARIABLE STIFFNESS PLYWOOD- PLASTIC PLATE

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ABSTRACT

A new optimization method of outer layer fibre directions and fiber relative concentrations of plywood plate with glass fibre-epoxy outer layers is proposed. The method minimizes structural compliance. It consists of two phases. The fibre directions are optimized in the first phase and concentrations in the second phase. The increase of stiffness is about 31% of the plate with optimized fibre direction and concentration comparing to a similar non-optimized plate.

Key words: plywood, glass fibre, fibre orientation, fibre concentration, minimal compliance

INTRODUCTION

Laminated plates and shells with variable stiffness have been intensively investigated during the past two decades.
These kinds of structures are becoming more popular due to the ability to achieve increased strength-to-mass and stiffness-to-mass ratios by tailoring the material properties. The fibre steering machines are becoming more popular in manufacturing variable stiffness glass or carbon fibre plates.
An optimal variable stiffness plate could be obtained by optimization of the fibre orientation angle (Keller, 2010; Pelletier, Vel, 2006; Gurdal, Olmedo, 1993) or thickness optimization (Almeida, Awruch, 2009; Muc, Muc-Wierzgon, 2012). A lamina with variable stiffness and curved fibres provides great flexibility to achieve the needed natural frequencies, mode shapes (Akhavan, Ribeiro, 2011), vibration amplitudes (Akhavan, Ribeiro, 2012) and buckling load (Setoodeh et.al., 2009). It is necessary to design constant thickness plates in many cases. Optimal properties of a constant thickness plate or shell are obtained by using the Genetic Algorithm (Sliseris, Rocens, 2011; Sliseris, Rocens, 2012) or Ant Colony algorithm (Sebaey et. al., 2011; Wang et.al., 2010; Hudson et. al., 2010) in cases of complicated objective function or many design variables. It is necessary to take into account the inter-laminar stress of variable stiffness lamina (Diaz et. al., 2012) in some cases.
The problem of optimal fibre orientation angle of multilayer lamina is successfully solved by using of the topology optimization approach (Diaz, Bendsoe, 1992; Bendsoe, 1989), discrete material optimization method (Lund, 2009; Niu et.al., 2010; Stegmann, Lund, 2005), Ant colony algorithm (Kaveh et.al., 2008) or Genetic algorithm (Hansel et.al., 2002). Optimizations of structural elements are done by taking into account uncertainty and nonlinear effects (Jung, Cho, 2004; Asadpoure et.al., 2011).
Flexural plates, like glass fibre reinforced polymer(GFRP)-plywood, with variable stiffness have not been investigated enough by now. The optimization method for this type of structure should be specially created. Therefore, this publication is proposing a new optimization method for GFRP-plywood lamina fibre direction and concentration optimization and providing some typical results.

OPTIMIZATION METHOD

The lamination parameters that define stacking sequence of lamina by 12 parameters are usually used in optimization because relationship between stiffness and lamination parameters is convex. The lamination parameters are related to each other, therefore the problem with feasible region always appears in the optimization procedure. As well as an extra procedure for stacking sequence rendering from lamination parameters is necessary. To simplify this optimization technique and make it more applicable to flexural plates with symmetrical layup the authors of this publication are proposing a new method.
This method is based on structural compliance minimization:

$$\min_{\phi,k} U^T(\phi,k)K(\phi,k)U(\phi,k), \quad (1)$$

where
$U(\phi,k) -$ displacement vector;
$K(\phi,k) -$ global stiffness matrix;

$\phi = \{\phi_1, \phi_2, \dots, \phi_{Ne}\}$- fibre orientation angles;

$k = \{k_1, k_2, \dots, k_{Ne}\}$- fibre concentrations, that is volume fractional part of fiber in GFRP layer.

This method directly optimizes the fibre orientation angle and concentration of only outer layers of symmetrical lamina. The outer layers play the most significant role in stiffness of flexural plate.

The material flexural stiffness matrix D_i of i-th finite element is modified by using the coordinate transformation matrix N and the fibre concentration coefficients k :

$$D_i = k_i N^T(\phi_i) D_i^0 N(\phi_i). \qquad (2)$$

The proposed method is based on the algorithm that is shown in Fig.1. The algorithm consists of three loops. The first loop runs until the convergence criteria are satisfied. The second loop goes through all finite elements from *1* to *Ne* (number of finite elements). The third loop goes through all discrete values of the fibre orientation angles from *1* to *N*.

The fibre orientation angles are changed by special procedure $R(x)=x_j$. This procedure changes the orientation angle to x_j in the region with the centre in *i*-th finite element and influence radius R_{inf}.

The finite element analysis is done inside all loops. The value of the compliance function *C(i,j)* (index *i* indicates *i*-th discrete angle and index *j* indicates *j*-th finite element) is calculated by using the results of the finite element analysis.

There is a special procedure that updates values of fibre orientation angles *x* inside the first loop. The updated value of *x* is obtained in each finite element according to minimal compliance.

The fibre concentrations are updated by using the following algorithm:

$$k_i = max\begin{cases} k_{min} \\ min\begin{cases} k_{max} \\ \left(k_i * \frac{C(i)}{L}\right)^{0.5} \end{cases} \end{cases}, \qquad (3)$$

where k_{min}, k_{max} −minimal/maximal value of possible concentration;

L −parameter that is used to limit the sum of concentrations in all finite elements.

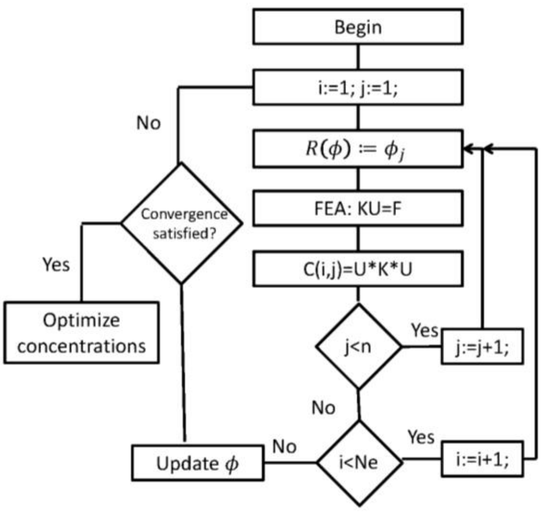

Figure 1. Fibre direction ϕ optimization algorithm

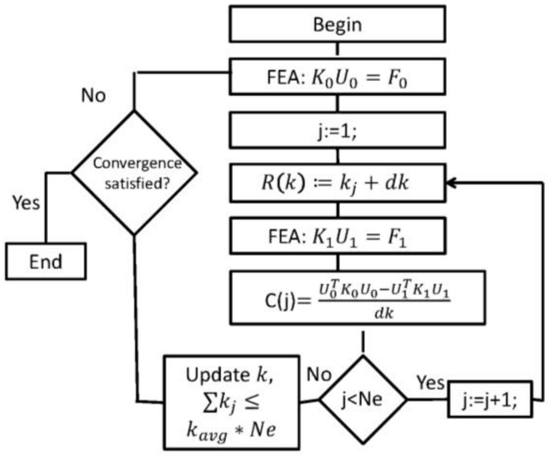

Figure 2. Fibre concentration k optimization algorithm

RESULTS AND DISCUSSION

The optimal fibre orientation angles and concentration ratio were obtained for a 19 layer symmetrical birch plywood sheet. The plywood sheet has the following lay-up $[\phi_i, 0, 90, \ldots, 90, 0, \phi_i]$. The total thickness of the sheet is 26 mm. The outer layer of the sheet was made of glass fibre- epoxy. The birch plywood was used with the following elastic properties (Sliseris, Rocens, 2012): $E_1 = 16400\ MPa, E_2 = 500\ MPa, G_{12} = 890\ MPa, v_{12} = 0.3$. The glass fibre of grade E was analyzed using the following elastic properties: $E_1 = E_2 = 85000\ MPa$, $G_{12} = 35420\ MPa, v_{12} = v_{21} = 0.2$ (Bank 2006). The epoxy glue was assumed to have the following elastic properties: $E_1 = E_2 = 3400\ MPa, v_{12} = v_{21} = 0.3, G_{12} = 1308\ MPa$ (Clarke, 2005).

Four discrete values of the fibre orientation angle were used: 0/45/90/135.

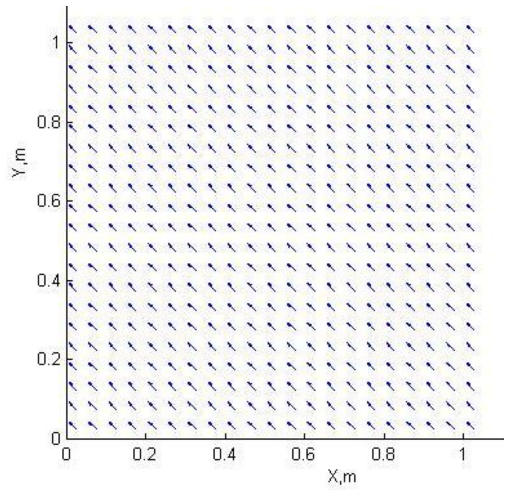

Figure 3. Fibre direction plot of a single span rectangular plate with dimensions 2.1 m x 2.1 m (due to symmetry shown one quarter of the plate)

The angle is between the x-axis (horizontal axis) and the fibre longitudinal axis. In all cases the influence radius (radius of domain where is changed fibre orientation angles) was constant R_{inf}=0.15(m).
The plates are loaded by 1 KPa uniformly distributed transversal load when searching for minimal compliance.

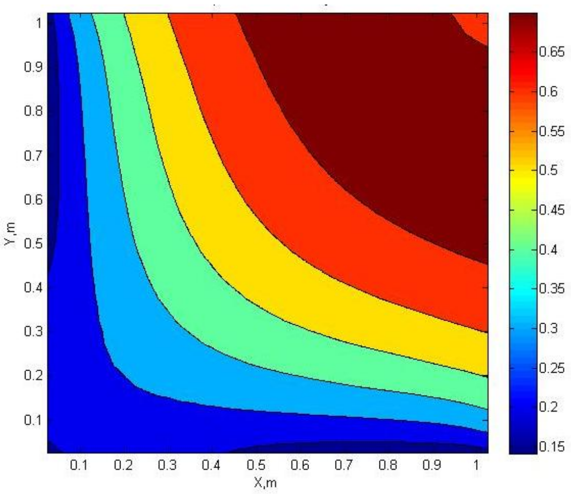

Figure 4. Fiber relative concentration plot of a single span rectangular plate with dimensions 2.1 m x 2.1 m (due to symmetry shown one quarter of the plate)

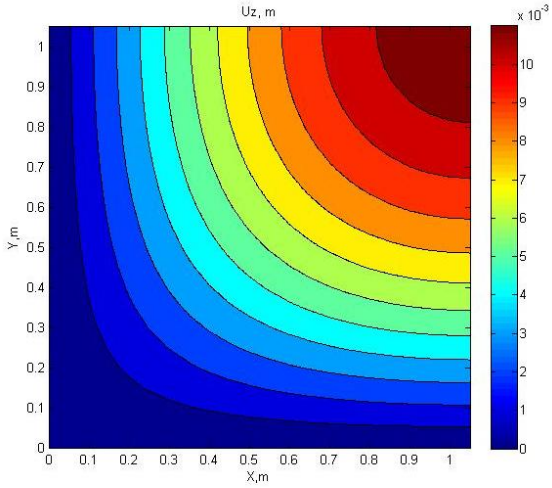

Figure 5. Deflection plot of a single span rectangular non-optimized plate with dimensions 2.1 m x 2.1 m (due to symmetry shown one quarter of the plate)

The optimal fibre concentration plot is shown in Fig. 4. It can be seen that the maximal amount of fibres should be put in the central part of the plate.
The maximal deflection of a non-optimized plate is 0.0117 m. The maximal deflection of a plate with optimized fibre directions and constant concentration is 0.0101 m. The maximal deflection of the plate with optimal fibre directions and concentrations is 0.008 m. The difference between

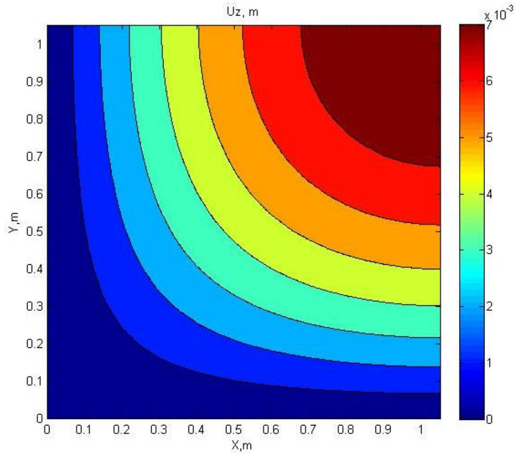

Figure 6. Deflection plot of a single span rectangular optimized plate with dimensions 2.1 m x 2.1 m (due to symmetry shown one quarter of the plate)

rotation angles of non-optimized and optimized plates is the same as for maximal deflection. The increase of stiffness of the optimized plate is not significant when a single span plate has one dimension significantly bigger than other.
The optimization was done also for three span plates. The plots of optimal fibre directions and concentration of plate 0.7 m equal spans in both directions as it is shown in Fig. 7 and Fig. 8. The fibre direction is orthogonal with support lines. Maximal fibre concentrations are necessary on support lines. The fibre relative concentrations in the middle of spans are about 25% less than on support lines.

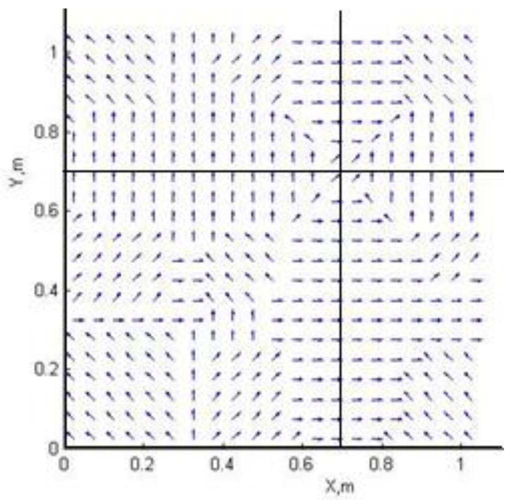

Figure 7. Fiber direction plot of a three span rectangular plate with spans 0.7 m x 0.7 m x 0.7 m in both directions (due to symmetry shown one quarter of the plate)

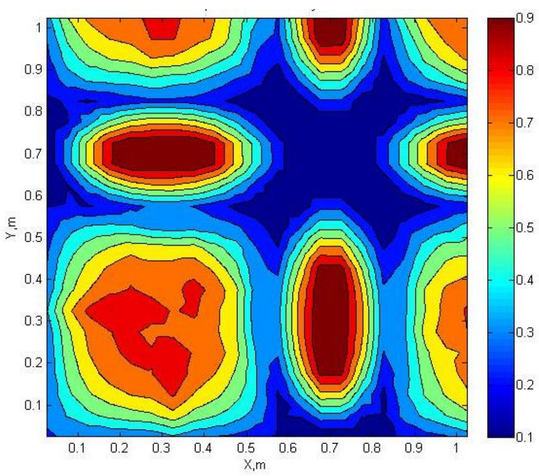

Figure 8. Fiber relative concentration plot of a three span rectangular plate with spans 0.7 m x 0.7 m x 0.7 m in both directions (due to symmetry shown one quarter of the plate)

The obtained optimal fibre orientation results show that a single simply supported plate could be effectively manufactured by making four different regions of the plate (see Fig. 3.). However, the concentration that is close to the necessary one could be achieved when additional four discrete regions are made (see Fig. 4.).

The three span plates could be divided in rectangular discrete domains to obtain fibre orientation and concentrations that are close to the necessary ones (see Fig. 7 and Fig. 8).

THE BEHAVIOR OF A PLATE WITH DISCRETE VARIABLE STIFFNESS

The manufacturing of continuously varying fibre placement may cause difficulties in manufacturing of GFRP-plywood composite. Therefore, the authors of this publication are proposing to make composite with discrete variable stiffness. It means that in discrete areas of the plate there are different fibre orientation and concentrations. Only discrete fibre orientation angles (0/45/90/135 degrees) and relative concentrations I (0.16/0.50/0.64/0.85); (0.18/0.50/0.67/0.85); (0.16/0.50/0.65/0.85) are used.

For each area the direction of glass fibres (0/45/90/135 degrees) and fibre relative concentration *I* are chosen. For each division three types of plates were analyzed – non-optimized directions (0 degrees) and non-optimized relative concentration (0.5); optimized directions (0/45/90/135 degrees) and non-optimized relative concentration (0.5); optimized directions (0/45/90/135 degrees) and optimized relative concentrations *I*.

Reduced modulus of elasticity (for relative concentration 0.5) for glass fibre layer is calculated (Bank 2006) E_1=58000 MPa; E_2=E_3=16400 MPa;G_{12}=G_{13}=6000 MPa;G_{23}=15000 MPa; ν_{12}= ν_{23}=0,233; ν_{13}=0.056

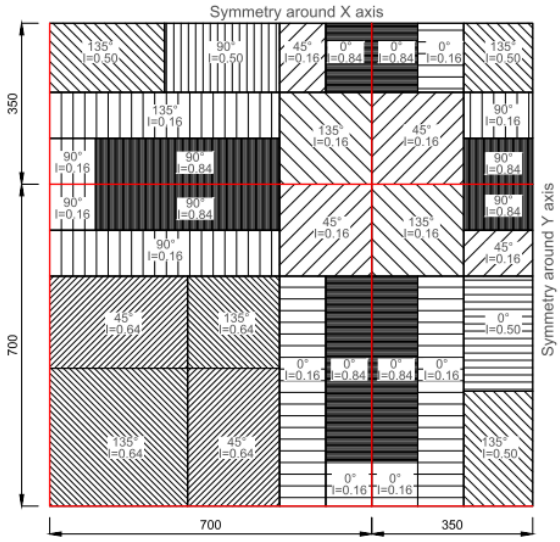

Figure 9. The plot of simply rectangular discrete domains including fibre orientations and concentrations of a three span rectangular plate (1 variant) with spans 0.7 m x 0.7 m x 0.7 m in both directions (due to symmetry shown one quarter of the plate)

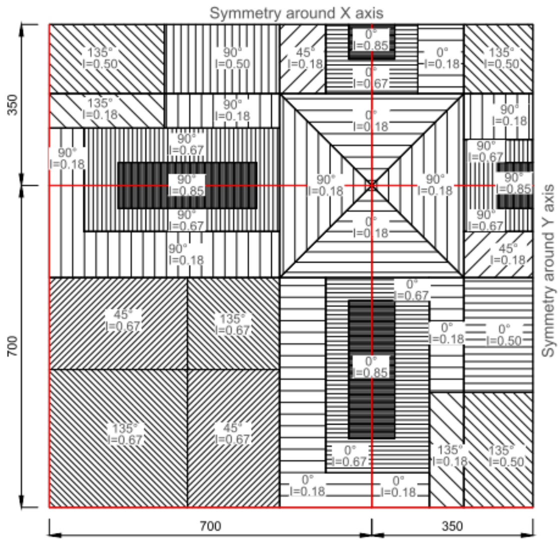

Figure 10. The plot of normal discrete domains including fibre orientations and concentrations of a three span rectangular plate (2 variant) with spans 0.7 m x 0.7 m x 0.7 m in both directions (due to symmetry shown one quarter of the plate)

The chosen discrete domains for 3 different complexities (1, 2, 3 variants) of discrete areas with intensities are shown in Fig. 9 - Fig. 11.

The plates are loaded by 1 kPa uniformly distributed transversal load the same as for one span plate example. Due to symmetry, one quarter of the plate was analyzed.

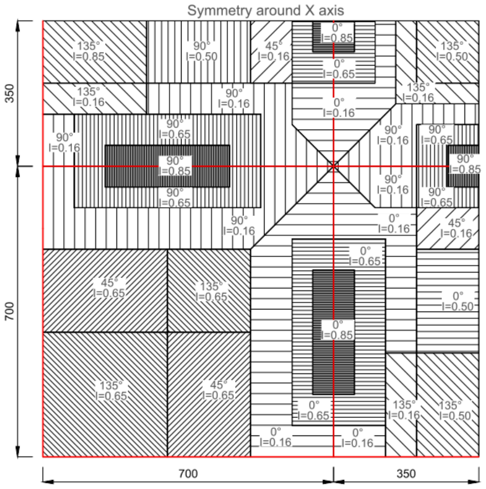

Figure 11. The plot of complex discrete domains including fibre orientations and concentrations of a three span rectangular plate (3 variant) with spans 0.7 m x 0.7 m x 0.7 m in both directions (due to symmetry shown one quarter of the plate)

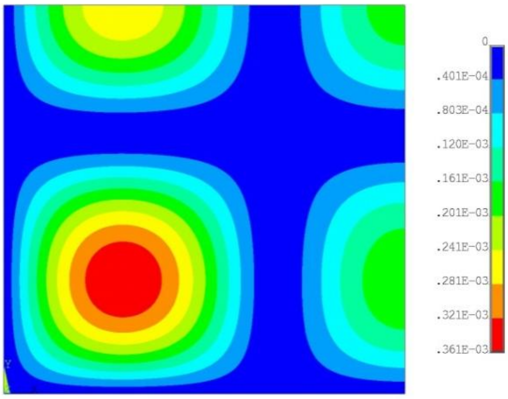

Figure 12. Deflection plot of a three span rectangular plate with spans 0.7 m x 0.7 m x 0.7 m in both directions for non-optimized plate (due to symmetry shown one quarter of the plate)

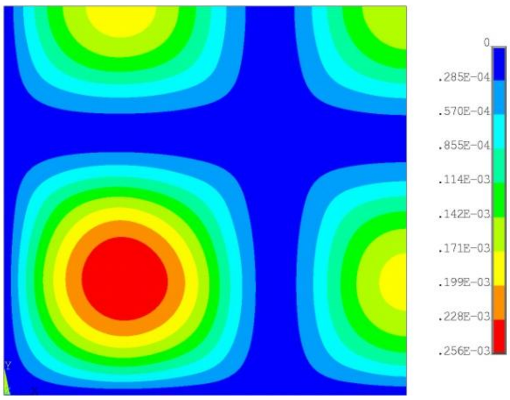

Figure 13. Deflection plot of a three span rectangular plate with spans 0.7 m x 0.7 m x 0.7 m in both directions for plate (1 variant) with optimized fibre directions and concentrations for simply rectangular division (due to symmetry shown one quarter of the plate)

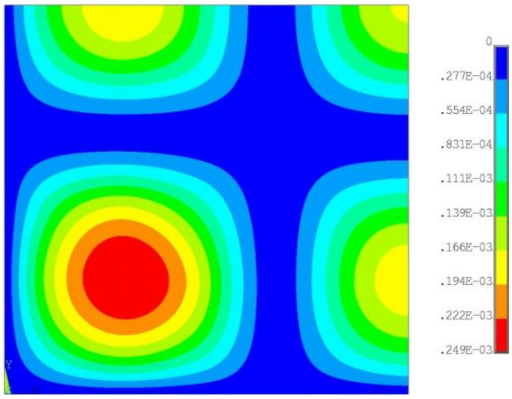

Figure 14. Deflection plot of a three span rectangular plate with spans 0.7 m x 0.7 m x 0.7 m in both directions for plate (2 variant) with optimized fibre directions and concentrations for normal division (due to symmetry shown one quarter of the plate)

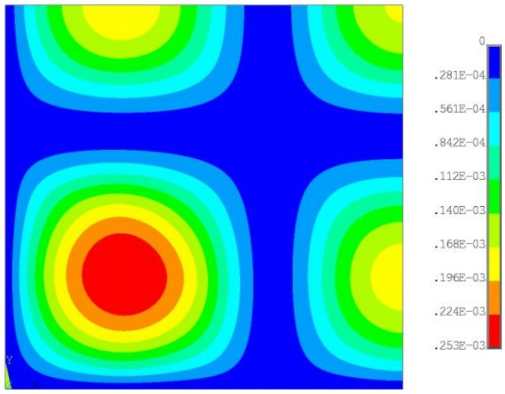

Figure 15. Deflection plot of a three span rectangular plate with spans 0.7 m x 0.7 m x 0.7 m in both directions for plate (3 variant) with optimized fibre directions and concentrations for complex division (due to symmetry shown one quarter of the plate)

The thickness of the GFRP layer is 1.75 mm, but the thickness of the plywood layers is 1.21 mm.

The maximal deflection of a non-optimized plate is 0.000361 m (see Fig. 12). For the plate with optimized fibre directions and concentrations the maximal deflection is 0.000256 m (see Fig. 13) for simply division; 0.000249 m (see Fig. 14) for normal division and deflection of the plate with optimal fibre directions and concentrations it is 0.000253 m (see Fig. 15). The difference between the rotation angles of non-optimized and optimized plates is the same as for maximal deflection.

The comparison between analyzed structures of plate is shown in Table 1.

It is found that differences of displacements between all three cases (type of division) are similar (decrease of displacements is 18% when only the fibre direction is optimized and 31 % when fibre directions and concentrations are optimized) that

shows that even simpler division may lead to decrease of displacements).
The maximum displacements were in the middle of the first and last span of the plate for the three span plate.
Stresses in the direction of glass fibres increase for about 12% (from 9 MPa to 10 MPa, see Table 2). It means that stress in plywood reduces (see Table 3).
The values of stresses reduce for 8% with optimization of fibre directions and 35% with optimization of fibre directions and concentrations for tension. But for compression stresses reduce for 6% with optimization of fibre directions and 16% by optimization of fibre directions and concentrations. All values for three types of divisions are shown in Table 3.

Table 1

Comparison of maximal deflection (m) for different variants

	var. 1		var. 2		var. 3	
Not optimized:	0.000361	0%	0.000361	0%	0.000361	0%
Optimized directions:	0.000295	-18%	0.000299	-17%	0.000299	-17%
Optimized directions and Intensities:	0.000256	-29%	0.000249	-31%	0.000253	-30%

Table 2

Comparison of maximal stress (MPa) in direction of glass fibre for different variants

	var. 1		var. 2		var. 3	
Not optimized:	8.960	0%	8.960	0%	8.960	0%
Optimized directions:	8.405	-6%	8.380	-6%	8.389	-6%
Optimized directions and Intensities:	9.970	11%	10.036	12%	10.002	12%

Table 3

Comparison of maximal stress in direction of wood fibre for different variants

	var. 1		var. 2		var. 3	
σ_{max} not optimized:	2.170	0%	2.170	0%	2.170	0%
σ_{max} optimized directions:	1.989	-8%	1.984	-9%	1.986	-8%
σ_{max} opt. directions and concentrations:	1.434	-34%	1.416	-35%	1.413	-35%
σ_{min} not optimized:	-1.387	0%	-1.387	0%	-1.387	0%
σ_{min} optimized directions:	-1.291	-7%	-1.310	-6%	-1.311	-5%
σ_{min} opt. directions and concentrations:	-1.179	-15%	-1.148	-17%	-1.169	-16%

CONCLUSIONS

New fibre direction and concentration optimization methods, which minimize the structural compliance, are proposed.
The GFRP-plywood composite plates provide a good possibility to increase the stiffness for more than 30%.
The maximal stress in the direction of glass fibre increases for 12%, but the maximal tensile stress in the direction of wood fibres decreases for 35 % for the optimized cases.
In future there should be a method created that calculates the optimal dimensions of discrete domains of the plate. Experimental investigations of GFRP-plywood plates should be made in future.

REFERENCES

Akhavan H., Ribeiro P. (2011) Natural modes of vibration of variable stiffness composite laminates with curvilinear fibers. *Composite Structures,* Vol. 93, No. 11, p. 3040-3047.

Akhavan H., Ribeiro P. (2012) Non-linear vibrations of variable stiffness composite laminated plates. *Composite Structures*, Vol. 94, No. 8, p. 2424-2432.

Almeida F.S., Awruch A.M. (2009) Design optimization of composite laminated structures using genetic algorithms and finite element analysis. *Composite Structures,* Vol. 88, No. 3, p. 443-454.

Asadpoure A., Tootkaboni M., Guest J. K. (2011) Robust topology optimization of structures with uncertainties in stiffness – Application to truss structures. *Computers and Structures,* Vol. 89, No. 11-12, p. 1131-1141.

Bank L.C. (2006) *Composites for Construction Structural Design with FRP Materials*. Jon Wilery & Sons. 567 p.

Bendsoe M. (1989) Optimal shape design as a material distribution problem. *Structural and Multidisciplinary Optimization,* Vol.1, No. 4, p. 193-202.

Clarke J.L. (2005) *Structural design of polymer composites*. London: E&FN SPON. 660 p.

Diaz A., Bendsoe M. (1992) Shape optimization of structures for multiple loading conditions using a homogenization method. *Structural and Multidisciplinary Optimization,* Vol. 4, No. 1, p. 17-22.

Diaz J., Fagiano C., Abdalla M.M., Gurdal Z., Hernandez S. (2012) A study of interlaminar stresses in variable stiffness plates. *Composite Structures*, Vol. 94, No. 3, p. 1192-1199.

Gurdal Z., Olmedo R. (1993) In-Plane Response of Laminates with Spatially Varying Fiber Orientations: Variable Stiffness Concept. *AIAA Journal,* Vol. 31, No. 4, p. 751-758.

Hansel W., Treptow A., Becker W., Freisleben B. (2002) A heuristic and a genetic topology optimization algorithm for weight-minimal laminate structures. *Composite Structures,* Vol. 58, No. 2, p. 287-294.

Hudson C. W., Carruthers J. J., Robinson A. M. (2010) Multiple objective optimization of composite sandwich structures for rail vehicle floor panels. *Composite Structures,* Vol. 92, No. 9, p. 2077-2082.

Jung HS., Cho S. (2004) Reliability-based topology optimization of geometrically nonlinear structures with loading and material uncertainties. *Finite Element Analysis and Design,* Vol. 41, No. 3, p. 311-331.

Kaveh A., Hassani B., Shojaee S., Tavakkoli S.M. (2008) Structural topology optimization using ant colony methodology. *Engineering Structures,* Vol. 30, No. 9, p. 2559-2565.

Keller D. (2010) Optimization of ply angles in laminated composite structures by a hybrid, asynchronous, parallel evolutionary algorithm. *Composite Structures,* Vol. 92, No. 11, p. 2781-2790.

Lund E. (2009) Buckling topology optimization of laminated multi-material composite shell structures, *Composite Structures,* Vol. 91, No. 2, p. 158-167.

Muc A., Muc-Wierzgon M. (2012) An evolution strategy in structural optimization problems for plates and shells. *Composite Structures,* Vol. 94, No. 4, p. 1461-1470.

Niu B., Olhoff N., Lund E., Cheng G. (2010) Discrete material optimization of vibrating laminated composite plates for minimum sound radiation. *International Journal of Solids and Structures,* Vol. 47, No. 16, p. 2097-2114.

Pelletier J. L., Vel S.S. (2006) Multi-objective optimization of fibre reinforced composite laminates for strength, stiffness and minimal mass. *Computers and Structures,* Vol. 84, No. 29-30, p. 2065-2080.

Sebaey T.A., Lopes C.S., Blanco N., Costa J. (2011) Ant Colony Optimization for dispersed laminated composite panels under biaxial loading. *Composite Structures,* Vol. 94, No. 1, p. 31-36.

Setoodeh S., Abdalla M. M., IJsselmuiden S. T., Gurdal Z. (2009) Design of variable-stiffness composite panels for maximum buckling load. *Composite Structures,* Vol. 87, No. 1, p. 109-117.

Sliseris, J.; Rocens, K. 2013. Optimal design of composite plates with discrete variable stiffness, *Composite Structures* 98 (April): 15-23

Sliseris, J., Rocens K. (2012) Optimization of multispan ribbed plywood plate macro-structure for multiple load cases, *Journal of Civil Engineering and Management* (accepted to publish).

Sliseris J., Rocens K. (2011) Rational structure of panel with curved plywood ribs. *in proceedings of ICBSE 2011 : "International Conference on Building Science and Engineering"*, April, p. 317-323.

Stegmann J., Lund E. (2005) Discrete material optimization of general composite shell structures. *International Journal for Numerical Methods in Engineering,* Vol. 62, No. 14, p. 2009-2027.

Wang W., Guo S., Chang N., Yang W. (2010) Optimum buckling design of composite stiffened panels using ant colony algorithm. *Composite Structures,* Vol. 92, No. 3, p. 712-719.