FOOTFALL INDUCED FORCES ON STAIRS

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ABSTRACT

To get a reliable dynamic performance of the modern flexible light-weight structures such as pedestrian bridges, flexible stairs, long span floors and tall public observation towers it is important to check the structure vibrations induced by human movement dynamic loads.

The paper gives some background information about recent advances determining the human induced dynamic forces. The paper presents a convenient method in order to obtain the equivalent continuous walking force histories and therefore essential parameters (dynamic load factors and corresponding phase shifts) that can be used in the structural design. The imperfectness of individual footfall forcing functions and differences between continuous walking force histories among individuals is taken into account. There are analyzed the experimental data obtained by using the inverse dynamic method (accelerometer technology) for walking load amplitude dependence on various pacing frequencies during the stair ascending or descending action.

Key words: dynamic load factor, footfall, human induced loading, stairs

INTRODUCTION

There are structures like slender stairs, footbridges, slender floors, grandstands or even light-weight observation towers where the most common source of the vibration is human activity (Kasperski et.al., 2011; Caetano, et. al., 2009; Feldmann et. al. 2009; Gaile, Radinsh, 2011; Gaile, Radinsh, 2012a; Straupe, Paeglitis, 2012). Increasing vibration issues for light-weight structures show that to perform design only based on the static loads is not enough anymore. Several recent extensive literature reviews and new guidelines highlight researchers interest in experimental identification and modeling human walking forces (Racic et. al., 2009; Zivanovic et. al., 2005; Venuti et. al., 2009; Butz et. al., 2008). The problem is the major part of these researches is done for human walking forces on flat surfaces. Still there is a little work done for studying the walking forces on stairs. The most relevant and recent on this subject is done by S.C. Kerr, N.W.M. Bishop and M. Kasperski (Kerr et. al., 2001; Kasperski et. al., 2011a). But researchers who applied the existing load models for the stair ascending or descending cases reported a very noticeable differences between the predicted and measured accelerations due to climbing (Davis et. al., 2009; Zhou et. al., 2011). In those works as a dynamic load for predictions seems to be applied only the vertical force component. However, for examples during the stair ascend the longitudinal force component amplitude is approximately 30% of the person's weight (Gaile, Radinsh, 2012b) and should be taken into account (Fig. 4). This indicates that loading models are still not complete and tuned properly. This correlates with the conclusion in the recent literature review done by V. Racic that disregarding the activity investigated, only the vertical ground reaction forces (GRF) on rigid surfaces for a single person are tested with modern non direct measurement technologies.

Riener (Riener et.al. 2001) in his experimental investigation found that ground reaction forces (GRF) were not significantly affected by the staircase inclination (tests were performed on the following stair inclinations: 24^0, 30^0, 42^0).

This paper presents a new and relatively easy method for obtaining the equivalent continuous walking force histories where the imperfectness of the individual footfall forcing functions are taken into account. It is done by utilizing the modern accele-rometery technology. There is presented the force amplitude dependence on the pacing rate during the stair ascending and descending case for all three components: vertical, lateral and longitudinal direction based on the experimental results. There are analyzed the walking harmonics that are mostly relevant in the structural design and parameters of the critical ones presented.

BACKGROUND

There are many different types of human activities such as walking, running, jumping and intentional swaying (vandal loading), that induce dynamic forces on structures. Except the vandal loading that is a provision of the accidental limit state according to so–called the limit state design code format (Eurocode 0, 1990), other activities mostly are associated with the comfort of the users of light-weight structures and therefore fall under the serviceability limit state. If the stairs are under consideration, the most frequent and therefore important dynamic loading is the stair ascending and descending action at human natural pacing rate. And there is a clear necessity to improve the existing load model of pedestrian induced forces to obtain better agreement between numerically calculated and experimentally measured structure response to human activities.

Experimental background

A lot of research is done on human ground reaction forces in the field of biomechanics (Ayyappa, 1997). The interest mostly is GRF values for distinct points and their chronological occurrence on the single foot step force time history (Gordon et. al., 2004). In the field of civil engineering dynamics there is an interest to simulate the continuous walking force histories that can be applied to the structure during the design process.
The common way to obtain the GRF is using the force platforms (Fig. 1).

Figure 1. Example of the force platform for GRF measurements

It is an instrumented plate installed flush with the ground to register GRF (Gordon et. al., 2004). Also the main results in the field of civil engineering dynamics regarding the GRF on stairs are obtained using this technology (Kerr et. al., 2001; Kasperski et. al., 2011b), where one of the steps being replaced with the force plate. Comparing to the single force plate, the treadmill technology allows analysis of many consecutive cycles over a longer period of time (Racic et. al., 2009) but it is suitable only for obtaining forces on flat or inclined surfaces. One of the most recent works to obtain experimental values of the walking force lateral component with the treadmill technology is done by Ingolfsson (Ingolfsson et. al., 2011). Both methods have a serious drawback because the measurement devices have a strong influence on human ability to move naturally.
Relatively a new concept to measure the GRF is using accelerometers that are capable of monitoring, storing, and downloading data of relatively small time intervals over a long period of time. Accelerometers are sensors that produce electrical signals proportional to the acceleration in particular frequency band and might be based on different working principles (Cunha et. al., 2008). The benefits of using accelerometers compared to more traditional gait analysis instruments include low cost, testing is not restricted to laboratory environment, accelerometers are small in size, therefore walking is relatively unrestricted and with an option of direct measurement of 3D accelerations (Kavanagh et. al., 2008). Another way with a great potential to obtain GRF is during the analysis to combine the visual motion tracking data recorded using cameras or sensors (Cappozzo, et. al., 2005) with known body mass distribution (Gordon, et. al., 2004). Both of these methods are so called the inverse dynamic methods and could be valuable in civil engineering applications to estimate the continuous human induced forces applied to the structure under a wide range of conditions.

Theoretical background

Theoretically the continuous walking force histories can be obtained by using kinematics of the motion of the human centre of gravity (COG) (Winiarski, et, al. 2009; Whittlesey, et. al., 2004) because dynamics of different parts of the body translate the center of gravity from one point to another in the most energy efficient way (Amin, 2012). The vertical walking force function can be obtained from a simple dynamic equilibrium based on the Newton's Second law (1).

$$F(t) = Mg + Ma(t), \tag{1}$$

where M is a body mass of the person, g – gravitational constant, a – acceleration of the COG.
COG or also known as a body center of mass (BCOM) represents the mean position of the total mass of human body as a multi-segment system (Racic et. al., 2009). The segmental masses and their centers can be found from different authors (Wu et. al., 2005; Vaughan et. al., 1999; Dumas et. al., 2006). This approach usually is used in the field of the inverse dynamics (Fig. 2).

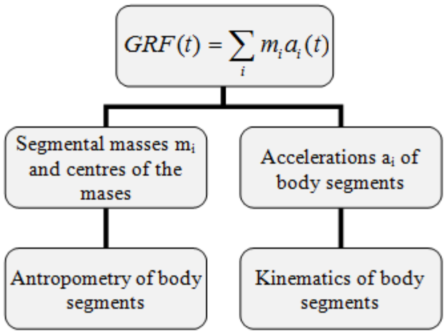


Figure 2. Flow chart of indirect measurement interpretation of human – induced loading

The drawbacks of the method are an incorrect assumption that body segments are rigid and placing markers or sensors accurately on the relevant segment of the body is problematic. As well the huge amounts of data due to the number of body segments under consideration are subject to errors. Therefore, for the civil engineering applications more convenient would be the placement of the sensor close to the actual COG of the whole body and not to the separate segments. It should be done tightly attaching the sensor on the body through the part that reduces the effect of the "soft tissue artifact (Leardini, 2005)". In the field of biomechanics it is known that approximate location of COG for women is 55% of height from the floor and 57% for men (Bartlett, 1997).

Although the probabilistic force models would be more suitable when simulating the walking forces as it is a stochastic narrow band process and depends on many parameters (Racic et. al., 2012) more convenient from the designing point of view would be a deterministic force model that takes into account non-periodicity of the force.

The most common way based on the Fourier decomposition for perfectly repeatable footfalls is to represent the walking force in the time domain as a sum of Fourier harmonic components, where the Fourier coefficient of the i^{th} harmonic often referred as the dynamic loading factor (DLF_i) is the base of this model. The example in Fig. 3 shows very significant scatter of the obtained DLF values of vertical force second harmonic (according to different authors). But the experimental measurements reveal that the character of the walking time history of different persons appears to be similar (Gaile, Radinsh, 2012b). Therefore, the authors suggest performing the averaging of individuals' continuous walking time histories instead of separate DLF_i values as it is usually done.

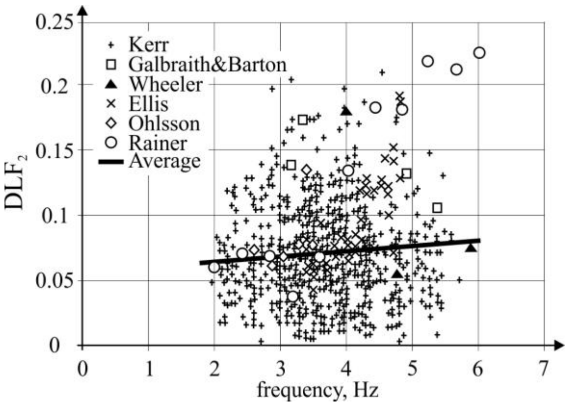


Figure 3. DLF values of second harmonic of walking force vertical component **(**Zivanovic, 2005)

EXPERIMENTAL INVESTIGATION

During the experiment to verify the proposed methodology for obtaining analytical expressions of human walking forces that took place in the Riga Technical University where accelerations of 22 persons' (mixed group of men and women) COG were measured and recorded as they ascended and descended the stair. Each participant of the experiment had several attempts with freely-chosen different pacing rates.

Two 3-axis light-weight (55g) USB accelerometers (Model X6-1A) were used to record the accelerations. The accelerometers were attached to the foam plastic light-weight boards tightly strapped to the COG horizontal axis of the subject in the front and back of the body (Fig. 4). The equipment is very light and does not vibrate independently from the individual's body. The measurement sample rate was 160 Hz.

Two flights of the stair (12 steps in each flight) were used to perform the test but only the second flight data, when the test subjects obviously were moving more naturally, was taken for the processing.

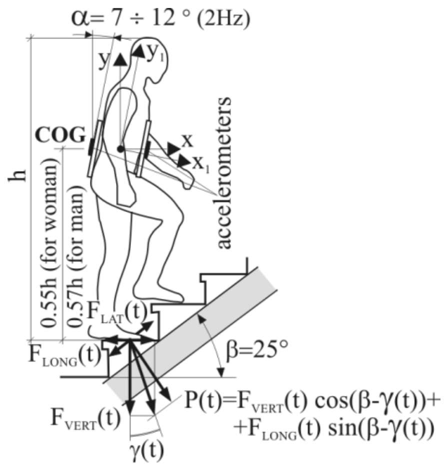


Figure 4. Illustration of the experimental setup

The experiment once again confirmed the sensitivity of the data when the test subject is under experimental conditions. The second flight measurements were always noticeably more stable and with smaller amplitudes. Additionally, some attempts during the experiment were completed with a laser streamer mounted on the front of the board lasing the staircase wall while a video camera recorded the change of the angle α from Fig. 4.

RESULTS OF THE RESEARCH

The presented method of obtaining the equivalent continuous walking histories that takes into account the imperfectness of the repeated footfall of the individual as well as differences between the individual walking force histories by averaging them is presented in Table 1. Consequently, it is possible to obtain the mean values of the DLF_i and the corresponding phase shifts. This is convenient to use in analytical or numerical calculations of the structure response under various human induced dynamic loading conditions.

To check the new method there were compared the S.C. Kerr's obtained results of the vertical force component DLF_i values with the new mean DLF_i values (for ascending (2Hz) and descending (2.15Hz) cases) and found to be in a very good agreement for the first harmonics. The results of the second harmonics slightly differ that correlate with the proposition of Davis (Davis et. al., 2009) to take a higher value for the second harmonic. The mean value obtained by the presented methodology is plotted on the S.C. Kerr's obtained results (Fig. 5).

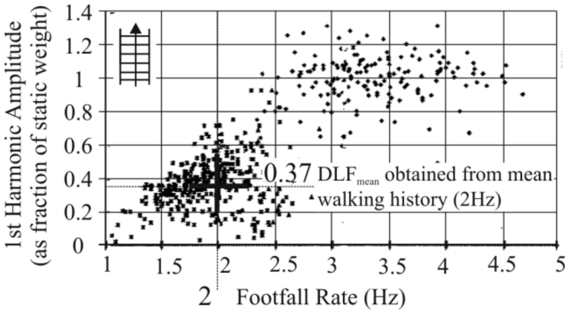


Figure 5. First harmonic amplitude of vertical force for ascending the stair

There are still no results to compare the longitudinal and lateral force component directions in the case of stair ascending/descending.

During the double averaging process the experimentally measured peaks smoothen and widen accordingly due to the lack of perfect periodicity between footsteps and individuals, therefore this effect on the response of the structure is investigated by numerical calculations. It is found that the error is less than 1% and therefore negligible (Fig. 6).

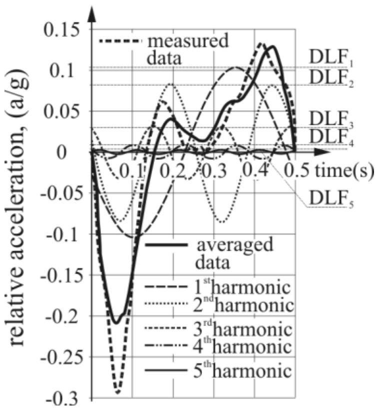


Figure 6. Walking force history averaging

The data in Fig. 7 and Fig. 8 that are obtained from the mean walking force history reveal differences between the stair ascending and descending process. The weight of the person was taken 740N. From the presented figures it follows that in contradiction to the vertical forces, the lateral and longitudinal components of the force reached the highest magnitudes during the ascending process that seems to be logical. A closer look at the middle part (small loops) of the figures reveals that during the ascending process a person tries to balance oneself in the lateral direction but during descending more in the longitudinal direction due to the inertia. The lack of the symmetry confirms that the presented method has taken into account the "leading leg" effect that especially becomes apparent for the stair ascending process.

To obtain the relationship between the walking force amplitudes and pacing frequencies it is suggested not to look at the separate DLF_i values corresponding to the relevant frequency but to take the mean value of the individual's experimental walking force history of *n* periods expressed as a range from maximum to minimum amplitude (2):

$$A = \frac{1}{n}\sum_{i=1}^{n} A_i^{poz} + \left|A_i^{neg}\right|. \qquad (2)$$

These relationships are obtained from the experimental data of the recoded walking histories of individuals and presented in Fig. 9 - 14.

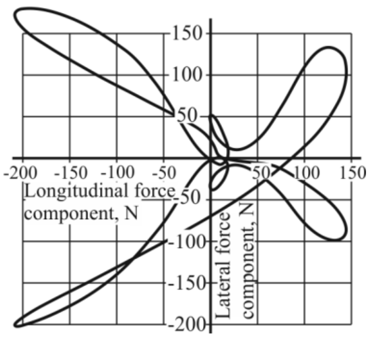


Figure 7. Path of the mean pedestrian force vector end point (ascending case at rate 2Hz)

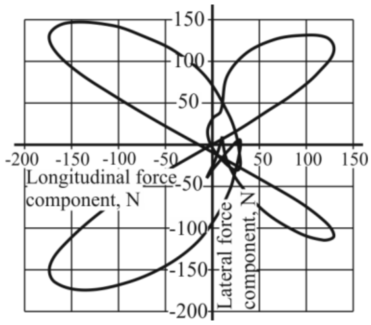
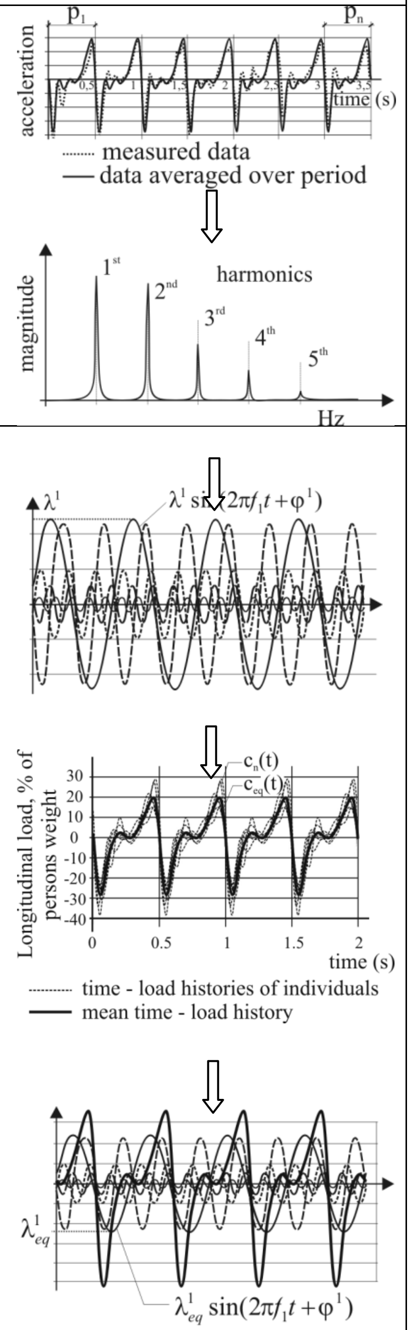


Figure 8. Path of the mean pedestrian force vector end point (descending case at rate 2,15Hz)

Table 1

Description of the new method for obtaining the equivalent continuous walking histories (analytical expressions)

Step Nr.	Action		Illustration
1.	Measure individual walking acceleration histories (*WAH$_i$*) and transform the recorded measurements from the local axis of the sensors into global directions taking into account the angle α in Fig. 4, then divide into periods p_n		(see figure below)
2.	Perform period averaging to obtain the equivalent period and further *WAH* of the person: $$p_{eq} = \sum_{i=1}^{n} p_n / n$$	(3)	
3.	Perform transformation from the time domain to the frequency domain via FFT (Fast Fourier Transform): $$p_{eq}(t) \Rightarrow p_{eq}(f)$$	(4)	
4.	Find the DLF value and relevant phase shift for each individual in order to obtain separate harmonics and further the analytical expression of individuals' walking force history *(WFH$_i$)*(5): $$c_n(t) = \sum \lambda^i \sin(2\pi f_i t + \varphi^i),$$ where i – order number of the harmonic, n – total number of contributing harmonics, λ^i – Fourier coefficient of the i^{th} harmonic often referred as the dynamic loading factor (DLF), f_i - i^{th} harmonic frequency (Hz), φ^i – phase shift of the i^{th} harmonics. If necessary, use the correction coefficient for magnitude to maintain the same area under the function as the experimental data have: $$\sum_n A_{ave} = \sum_n A_{\exp}$$	(5) (6)	
5.	Perform averaging between the functions of *WFH$_i$* to obtain the equivalent (mean) continuous walking time history: $$c_{eq} = \sum_{i=1}^{n} c_n / n$$	(7)	
6.	Transformation from the time domain to the frequency domain via FFT: $$c_{eq}(t) \Rightarrow c_{eq}(f)$$	(8)	
7.	Find the DLF value (λ^i_{eq}) and relevant phase shift of mean walking history in order to obtain separate harmonics and further the analytical expression of the mean walking force history in vertical (9) and lateral or longitudinal directions (10): $$F_y(t) = G + \sum_{i=1}^{n} G\lambda^i_{eq,y} \sin(2\pi f_i t + \varphi^i_y),$$ $$F(t) = \sum_{i=1}^{n} G\lambda^i_{eq} \sin(2\pi f_i t + \varphi^i),$$ where G is a static weight of the subject's body (N)	(9) (10)	



Each of the experimental points on the chart is the mean value of 12 periods (number of steps in the stair flight). The vertical dynamic walking forces are sensitive to the pacing frequency until the point of noticeable scatter in the values of each test subject and therefore the mean value might be taken as constant. The shape of the mean value function is similar to other researchers' works. The longitudinal and lateral force component is sensitive to changes of the pacing frequency with a tendency to increase by adding the walking speed in the case of the stair ascending. For the descending case these components seem to be very slightly dependent on the persons' pacing frequency and therefore could be regarded as constant.

If assumed that changes in the force amplitude due to different pacing frequencies divide proportionally between the harmonics and therefore DLF values, it is possible to find relationship that describes each of the relevant DLF value dependence from the person's pacing frequency (Table 2). As an example, there were calculated DLF values for the pacing frequency 1.6Hz and compered to the Kerr data.

Most of the data correlate very well except the second harmonic for the descending case. By analyzing the mean values obtained by Kerr (in Figure 18 (Kerr et. al., 2001)), it follows that the second harmonic does not depend on the pacing frequency for the stair descending case and therefore the shape of the walking force history should differ dramatically even if there is a slight change in the walking speed. Apparently, it is due to the rapid change of the first harmonic values. However, this does not appear in the experimental data for fundamental pacing frequency range of *1Hz* $\leq f \leq$ *2.3Hz*. On the other hand, Kerr in his paper states that "…the weight is transferred quickly from one leg to the other, which creates a deeper hollow between the humps. The greater the distance between the hollow and the humps, the greater the second harmonic". Therefore, the second harmonic amplitude should change if the pacing frequency changes for the stair descending or ascending case.

This example demonstrates the advantage of the new method based on the averaging of the continuous walking histories and not the separate DLF values obtained from one or few steps. Also to get a correct analytical function of human induced forces, the phase shift of each harmonic is an important parameter. By averaging only DLF values this information is lost. Another aspect is that the averaging of particular Fourier coefficient between the different Fourier series will not necessarily give the average forcing function due to the connection between the coefficients in each of the function.

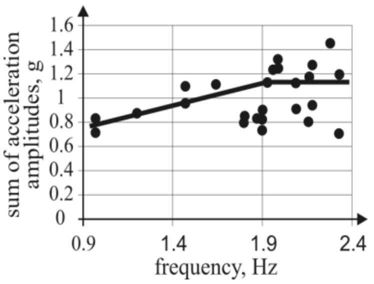


Figure 9. Relationship of amplitude and pacing frequency (vertical direction, ascend)

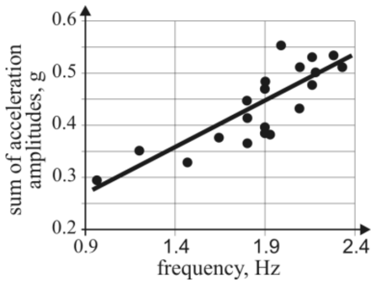


Figure 10. Relationship of amplitude and pacing frequency (longitudinal, ascend)

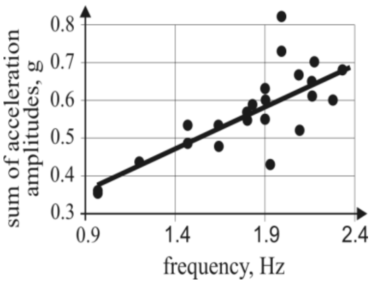


Figure 11. Relationship of amplitude and pacing frequency (lateral, ascend)

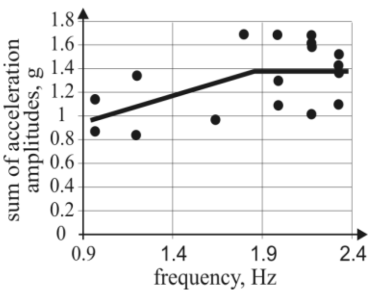


Figure 12. Relationship of amplitude and pacing frequency (vertical direction, descend)

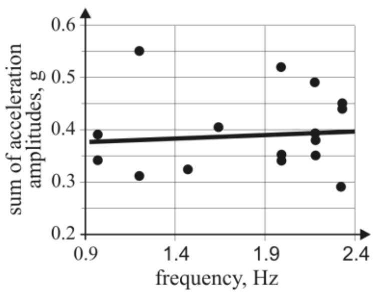


Figure 13. Relationship of amplitude and pacing frequency (longitudinal, descend)

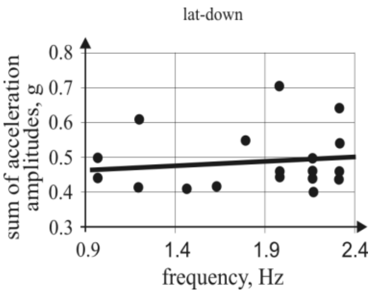


Figure 14. Relationship of amplitude and pacing frequency (lateral, descend)

Table 2

DLF values of stair ascending and descending dominant harmonics

Action	Proposed $DLF_n(f)$	Calculated DLF_i at *1.6Hz*	Average DLF by Kerr at *1.6Hz* (Kerr et. al., 2001)
Ascending, vertical	$DLF_n(2Hz)\cdot(0.94f-0.88)$; $1\le f<1.95$ $DLF_n(2Hz)$; $1.95\le f\le 2.3$	$0.23(DLF_1)$ $0.13(DLF_2)$	$0.27(DLF_1)$ $0.12(DLF_2)$
Descending, vertical	$DLF_n(2Hz)\cdot(0.99f-1.13)$; $1\le f<1.85$ $DLF_n(2.15)Hz)$; $1.85\le f\le 2.3$	$0.27(DLF_1)$ $0.06(DLF_2)$	$0.24(DLF_1)$ $0.22(DLF_2)$
Ascending, longitudinal	$DLF_n(2Hz)\cdot(1.49f-1.98)$; $1\le f\le 2.3$	$0.049(DLF_1)$ $0.044(DLF_2)$	- -
Descending, longitudinal	$DLF_n(2.15Hz)$; $1\le f\le 2.3$	$0.07(DLF_1)$ $0.1(DLF_2)$	- -
Ascending, lateral	$DLF_n(2Hz)\cdot(2.2f-3.4)$; $1\le f\le 2.3$	$0.012(DLF_1)$ $0.013(DLF_3)$	- -
Descending, lateral	$DLF_n(2.15Hz)$; $1\le f\le 2.3$	$0.08(DLF_1)$ $0.11(DLF_3)$ $0.07(DLF_5)$	- -

where f – pacing frequency, Hz;
n – number of the harmonic;
$DLF_n(f)$ - dynamic load factor at pacing frequency f for the harmonic n;
$DLF_n(2Hz)$ - dynamic load factor at pacing frequency 2Hz and corresponding phase shifts for the harmonic *n* found from Table II – IV (Gaile, Radinsh, 2012b).

CONCLUSIONS

The presented new method of obtaining analytical functions of continuous walking histories is based on experimentally obtained continuous walking histories of individuals that are found by the inverse dynamic method and uses kinematics of the human center of gravity. Instead of the traditional approach when relationship between the walking pace and force amplitudes is based on the average harmonic (DLF) amplitude, this method proposes averaging between continuous walking histories priory proceeded in order to take into account the imperfections of repeated footfall. In this way there is preserved information about the phase shifts – necessary parameter to obtain analytical function based on the Fourier series. The main advantages of using the presented method are as follows:

1) Possibility to estimate continuous human–induced forces of different actions applied to the structure under a wide range of the conditions due to the non-laboratory restrictions;
2) The measurement devices do not have a strong influence on human ability to move naturally;
3) Requirement of a low cost instruments: few accelerometers capable of storing and downloading data with relatively small time intervals;
4) Allows to obtain not only dynamic load factors but also the phase shift values associated with the mean walking history;
5) The obtained analytical mean function contains information about imperfections of the person's footfalls and differences between the continuous walking histories but still it is a deterministic force model. Unlike the probabilistic force models it is more convenient to handle when performing analytical or numerical calculations of the structure under consideration.

To test the method there were obtained equivalent DLFs and their dependence on the walking pace for all three force directions (the stair ascending and descending case): vertical, longitudinal and lateral. Vertical results are compared with the measurements of the vertical component done by

Kerr using the force plate technology. The overall results correlate very well except the second harmonic were the Kerr's data have a very significant scatter and the mean value does not depend on the walking pace (stair descending case) that seems not to be quite realistic.

Descending the stair produces higher vertical force amplitudes than ascending that is logical and in agreement with other researchers' works. The lateral and longitudinal direction force amplitudes strongly depend on the walking pace only in the case of the stair ascending. In the case of the stair descending these might be considered as constant but with smaller amplitudes. The authors are not aware of any information that could be compared with the obtained results for these two directions. Recent concerns about some of the light-weight public observation towers excessive vibrations and dissatisfaction of the visitors' comfort criteria call for greater attention to longitudinal and lateral force components during a long stair ascending or descending process.

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