EVALUATION OF MATHEMATICAL MODEL THE TEMPERATURE REGIME OF MILKING PARLOR
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Abstract. Mathematical model of the dynamic resemblance of the inside air temperature that could be solved by the simple method has been made. The heat accumulation of the barrier and equipment, periodical heaters and the flow of the animal heat have been estimated with the help of this model. The model parameters have been chosen after the analysis of the technological and technical data of the typical milking parlors. When the outside temperature is 20 °C below zero, and the inside temperature during milking time should be 15 °C, then the heater has to be used twice a day for 3 hours, and its power should be 1.5 kW for every milking place. This has been calculated by use of this mathematical model. The inside temperature fell down to 6 °C between the milking periods.

Key words: accumulation, ventilation, heating, animal heat, temperature, dynamics, model.

Introduction
Dynamic thermal processes take place in the milking parlor. During the milking time (most often twice a day for two-three hours) cows separate a lot of heat. Milking parlors have often installed constant or periodical operating heat sources. The optimum temperature of the air during milking time is approximately 18 °C, and during the winter period the temperature should be not less than 10 °C. When there are no people working in the milking parlor the temperature can be still less, but it should be positive for the water to stay in liquid state and for the proper operation of electronic apparatus. The fluctuations of the heat flows in the premises have been influenced by the temperature changes of the barrier surfaces and the thermal energy accumulated in barrier and the one returned to the premises.

There are methods to anticipate the microclimate indices of the premises and to substantiate the factors governing these indices during the stationary exchange of heat and moisture. The methods have been also created to analyse the stationary and dynamic heat exchange in the external barrier of the buildings (Iljinskij, 1974). Recently the sound mathematical models of the balance similarity of the pigsty temperature, water vapour, dust, ammonia in unstationary situation (Zhang et al., 2001), the dynamic and stationary processes of heat and water vapour in the pigsty (Daskalov, 1997) have been investigated. But the methods prepared have not estimated the dynamics of the heat accumulation of the building barrier and the equipment, and the heat flows. The calculation of the heat accumulation would allow to more fully estimate the required output of the heaters and to foresee the dynamics of the air temperature of the premise. Besides the models used are differential equations that have been solved with special Simulink program (Zhang et al., 2001).

The work aim is to make and solve the mathematical model of the dynamic similarity of the air temperature inside the building. This model enables to estimate the heat accumulation in barrier and equipment, and the periodical flow of the heating and animal heat.

Methods
The thermal balance of the premise estimating the heat accumulation by the barrier and equipment and periodical heating can be expressed by the following differential equation:

\[
\sum Q_{ac} \frac{dt}{d\tau} = \sum Q_a + \sum Q_w - \sum Q_d (t_i - t_0),
\]

(1)

where \(\sum Q_{ac}\) – the heat amount accumulated and released by the barrier and equipment, when the surface temperature changes by one degree, kWh/K;
\(\sum Q_a\) – the sensible heat flow of the animals, kW;
\(\sum Q_w\) – the heating intensity, kW;
\(\sum Q_d\) – the flow of the heat losses via the external barrier together with the ventilation air, when the temperature difference between the inside and outside air is one degree, kWh/K;
\( t_i, t_0 \) – the temperature of inside and outside air, °C;
\( t \) – the temperature of barrier and equipment surface, °C;
\( \tau \) – the time, h.

We chose the particular peculiar technical and thermal indices of the milking parlors, such as, the area of barrier, the mass of equipment, ventilation intensity, falling to one milking place, that in its turn heat one kW of the total heat flow from the cows. All the terms of equation (1) divided by the total heat flow of the animals will give:

\[
\frac{Q_{ac}}{W} \frac{dt}{d\tau} = \varepsilon + Q_u (t_i - t_0),
\]

where
\( Q_{ac} \) – the specific amount of accumulated and released heat, i.e. the heat amount accumulated and released by the barrier and equipment, falling to the unit of the total heat flow from the cows, when the surface temperature changes by one degree, kWh/(kW·K);
\( \varepsilon \) – the ratio of the sensible heat of cows with the total heat;
\( Q_u \) – the specific heating intensity, i.e. the heat flow of the heaters falling to the unit of the total heat flow from the cows, kW/kW;
\( Q_d \) – the specific flow of the heat losses, i.e. the flow of the heat losses via the external barrier and that with the ventilation air falling to the unit of the total heat from the cows, when the difference between the inside and outside air temperatures is one degree kW/(kW·K).

To solve this differential equation (2) both variables should be placed in different sides of the equation. Besides, we will chose that the surface temperature of the barrier and equipment has been equal to air temperature, and when the time changes from 0 to \( \tau \), then the inside temperature will change from \( t_{i0} \) to \( t_{i\tau} \). Then

\[
-\int_0^{\tau} \frac{d\tau}{Q_{ac}} = \int_{t_{i0}}^{t_{i\tau}} \frac{dt}{Q_u (t_i - t_0) - \varepsilon - Q_u}.
\]  

After the rearrangement of equation (3), we get such equation for calculation the dynamic relationship of the inside air temperature and the specific thermal and technical indices of the premises and time

\[
t_{i\tau} = t_{i0} \exp\left(-\frac{Q_d}{Q_{ac}} \tau\right) + t_0 \left[1 - \exp\left(-\frac{Q_d}{Q_{ac}} \tau\right)\right] + \varepsilon + \frac{Q_u}{Q_d}.
\]

When there are no cows in the milking parlor, \( H = 0 \).

Equation (4) should be changed so that it would be much easier to calculate the required heating intensity.

When the difference between the inside and outside air temperature is \( t_{i1} - t_0 = \Delta t \), and the permissible fall of the inside air temperature is \( t_{i\tau} - t_{i1} = \Delta t_i \), then the required specific heating intensity will be

\[
Q_w = \frac{Q_d}{1 - \exp\left(-\frac{Q_d}{Q_{ac}} \tau\right)} \frac{\Delta t + \Delta t_i}{\Delta t} - \varepsilon.
\]  

When there are no cows in the milking parlor, \( \varepsilon = 0 \).
The amount of the specific accumulated and released heat of the barrier and equipment when the temperature change period has been 12 hours is

\[ Q_{ac} = 10^{-3} \left( 0.728 A_{ac} \sqrt{c_a \lambda \rho} + 0.278 c_m M \right), \]  

where \( A_{ac} \) – the area of the barrier that accumulate the heat, falling to the flow unit of the total heat flow from cows, \( m^2/kW \);
\( c_a \) – the specific thermal capacity of the barrier, \( kJ/(kgK) \);
\( \lambda \) – the coefficient of the thermal conductivity of the barrier, \( W/(mK) \);
\( \rho \) – the density of the barrier, \( kg/m^3 \);
\( c_m \) – the specific thermal capacity of the equipment, \( kg/(kgK) \);
\( M \) – the mass of the equipment falling to the unit of the total heat flow of the cows, \( kg/kW \).

The flow of the specific heat losses is

\[ Q_d = g c_o + x, \]  

where: \( g \) – the specific ventilation intensity, i.e. the amount of air calculated for the unit of the total heat of the cows, \( kg/kJ \);
\( c_o \) – the specific thermal capacity of the air, \( kJ/(kgK) \);
\( x \) – the module of the heat losses via the external barrier, i.e. the flow of the heat losses via the external barrier falling to the unit of the total heat flow of the cows, when the difference between the inside and ambient air is one degree, \( 1/K \).

Module

\[ x = \frac{10^{-3} \left( \sum UA + \psi P \right)}{\sum Q_0}, \]  

where: \( U \) – the coefficient of the heat transmission of the external barrier, \( W/(m^2K) \);
\( A \) – the barrier area, \( m^2 \);
\( \psi \) – the specific heat losses via the floor and foundations, \( W/(mK) \);
\( P \) – the perimeter of the foundations, \( m \);
\( \sum Q_0 \) – the flow of the total heat of the cows, \( kW \).

Mathematical model of the differential equation (2) of the inside air temperature that could be solved by the simple method (Equation 4) has been made. The heat accumulation of the barrier, periodically switched on heaters and the flow of the animal heat have been estimated by the help of this model. Equation (6) is used to calculate the required intensity of heating.

**Results and discussion**

After the analysis of the data of technological and technical parameters of the milking parlors and the physical data of the construction structures, the data of the model parameters has been received, that are defined as follows:

- one milking place is adequate to 1 kW of the total heat flow of the cows;
- two walls of the milking parlour are external, and two are internal. The internal layers of all barrier accumulate and release the heat. The thermal losses occur via external walls, ceilings and floors (foundations);
- the coefficient of heat transmission, \( (W/m^2K) \), of the walls is 0.5 and the ceiling is 0.25. The specific heat losses via the floors and foundations are 0.9 \( W/(mK) \);
- the milking of cows has been twice a day from 6 to 8 a.m. and from 6 to 8 p.m.;
- during winter or cold season the heater has been used one hour before the milking time and is switched off after the milking has been finished;
- the ambient air temperature is 20 \( ^oC \) below zero;
- the specific ventilation intensity is 0.034 \( kg/kJ \) during the milking and 0.017 \( kg/kJ \) between the milking.
The calculation results of the analysed model have been given in the Figure 1.

Fig. 1. Dynamics of air temperature of the milking parlor when the ambient temperature is 20 °C below zero:

- $c = 0.55$ – during the milking;
- $Q_{w} = 1.5 \text{ kW/kW}$ – from 1 hour before the milking to end of the milking

The calculation results of the model show that when the ambient air temperature is 20 °C below zero, the temperature in the milking parlor at the beginning of milking should be 10 °C, thus the heater should be switched on one hour in advance (the heater output should be 1.5 kW for one milking place). Then the temperature of the milking parlor would be 15 °C at the end of the milking. After the milking has been finished and the heater switched off, the temperature will fall down to 6 °C.

Conclusions

The model of dynamics resemblance of the air temperature has been made. This model enables to estimate the heat accumulation by the barrier and equipment, and the periodical heat flow of the heater and the cows. It has been suggested to express the model parameters via the total heat flow of the animals, when the milking place is adequate to 1 kW of the total heat flow. When the ambient air temperature is 20 °C below zero and we expect to raise the inside air temperature up to 15 °C, then the utilization of this model has enabled to define that: the heater should be switched on twice a day for the period of 3 hours. The heater output should be 1.5 kW for one place. The inside air temperature decreases to 6 °C between the milking periods.

References