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GRAPH THEORY AND AGROBUSINESS

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Abstract. Nowadays the decision making process within changing condition is a crucial issue in success of the company. A practical tool for decision support is the graphical model reflecting the decision problem of its structure, to formulate and to obtain the exact mathematical solution in the form of precise assessment. The aim of the paper is to present characteristics of the role of this theory in agribusiness enterprises and its impact on the agricultural sector. In the research paper, the descriptive and comparative methods were used. The paper presents general overview of the importance of the graph theory and nets. The authors underline that it is a crucial issue for many areas of science. Scientific interest in networks has a long tradition and is associated with the emergence of graph theory in mathematics. Another area of research for graphs and networks is social network analysis. The transfer of network theory to the field of logistics is related to the companies striving to develop supply, production and distribution, and to increase the effects of cooperation between companies. Regardless of organizational and legal conditions, enterprises become participants in so-called logistics networks. In order to explain the concept of logistic network, the authors have used - graph theory . The authors' research has shown the application of the theory in agribusiness.

Key words: graphs and nets, agricultural sector, business.
JEL code: A12, G14, R40

Introduction

To ensure the success of the company by making rational decisions in constantly changing conditions, there are set high requirements for the persons responsible for the management. The management process, irrespective of its subject epitomized is associated with the decision-making process, and in the effective management of the taken decisions must be optimal. Graphs and networks as a tool for decision support are the graphical model reflecting the decision problem of its structure, to formulate and to obtain the exact mathematical solution in the form of precise assessment. Field of knowledge based on the theory of graphs and networks is a significant chapter of the operations research (Wojciechowski J Pienkosz K., 2013).

The use of information system based on the theory of graphs in the organization has to increase its competitiveness. Information plays a crucial role in all markets. It gives many advantages: increased production of shortening the delivery time to increase sales. Logistics systems in agribusiness are defined as systems participating in the time-spatial transformation of material goods, while the processes taking place in them is logistics. Logistical processes are associated with both the flow of materials and information from the providers of factors of production to their users, together with disposal, resulting in a process of waste. Logistical process should be understood as an ordered chain of all operations involved in the broad movement of different material (Parlinska, M., 2008).

The problem of bilateral associations with the use of the language of graph theory and an indication of possible applications in the area of search and match of job seekers and employers was also the area of scientific research (Koziol-Kaczorek D., Pietrych L., 2016).

The aim of the paper is to present the importance of the graph theory and nets in agricultural economic sector. The in the research, descriptive and comparative methods as well graph theory were implemented. Information and data were taken from literature papers, investigations and calculations made by authors.

Authors have tried to answer the following research questions.

1) What is the basis for shaping supply chains?

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2) What is the role of graph theory?

3) How the graph theory can be implemented in agriculture sector?

There are many practical real-life logistics' problems, for example: mail delivery, garbage collection, salt gritting on icy roads during the winter, street cleaning in the cities and villages and many others. To achieve the intended goal, selected assumptions of graph theory and their use in logistics of the agribusiness sector were presented.

Definitions and basic concepts used in the research

Graph theory as a division of mathematics concerned with the study of their properties is an important mathematical tool used in many different areas, such as computer science, calculus, economics, management, sociology or logistics in agriculture. Creating algorithms for designating certain properties of graphs is one of the important areas for action. These algorithms are used to solve many practical tasks (Wilson R., 2012; Bronsztejn I.N. et al.,2013).

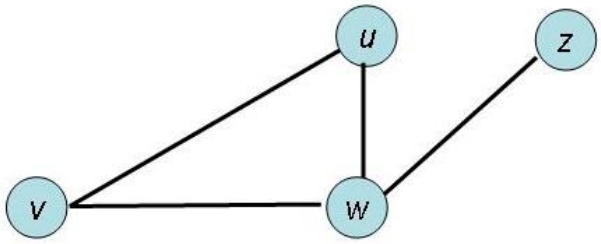
A simple graph G defines the two ordered sets: G = (V; E), where V is any non-empty finite set whose elements are called vertices (or nodes), while E is a subset of the set of all 2-element collections whose elements belong to V called edges.

Collection of V is called a collection of vertices, a set E of edges of the graph G. We say that the edge of the {E} connects vertices v and w, we mark it a symbol of vw. The elements of the set of vertices V and edges E in graphs are located logically, or form a relationship (R). Formal mathematical writing may be the following $V \neq \varnothing$, $E \subseteq V$. For graphs, when we will be talking about V, it will mean a finite collection of points forming the vertices of the graph.

Notation V = {v1, v2, ..., vn}, and (E) is a set of connections between these points.

Let will be chosen the set E = {a1, a2, a3, ... an}, where the existence of (aij) means the existence of edge between node vi with the vj.

For example, the figure represents a simple graph G, the set of vertices V (G) is a set of {u, v, w, z} and the set of edges E (G) consists of the edges, uv, uw, vw and wz.



Source: author's calculation

Fig. 1. **A simple graph G with vertex set V (G) and edge E(G)**

There are many definitions in the theory of graphs, but for this paper there are presented only some chosen from them.

1) The edge (i, j) is called a combination of vi, vj with regard to orientation (from vi to vj). The degree of the vertex is the number of edges whose end is the given vertex.

2) A road is a route where no vertex occurs more than once.

3) The loop is an edge whose beginning and end is the same vertex. Graphs with cycles and loops are called cyclic graphs, where cycles and loops do not occur, they are called acyclic graphs.

4) Graphs are called oriented (or directed), where there is a relationship (R) and the connections between nodes are directed and the order of occurrence in time individual vertices is specified.

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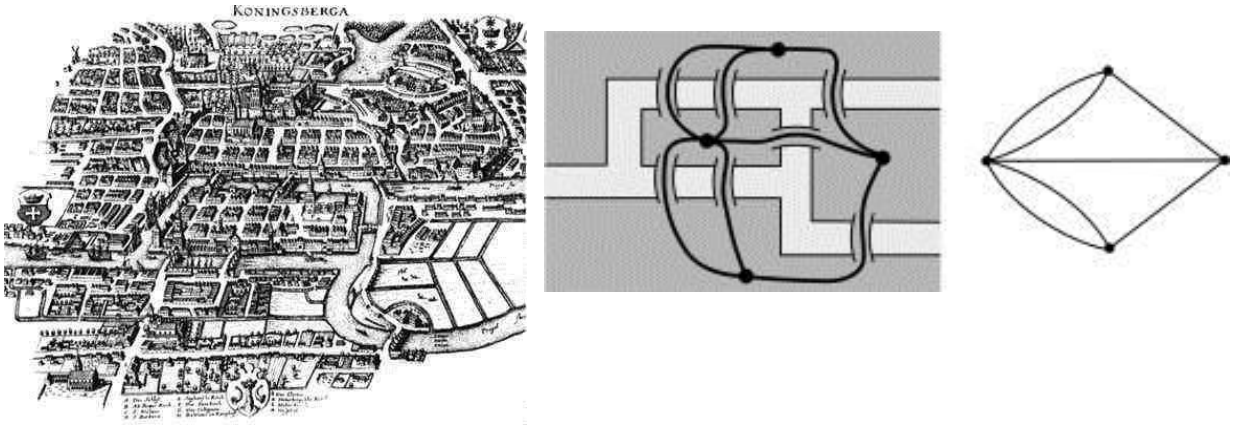
5) The graphs are not-oriented, when the connection between the vertices exist without taking into account the orientation.



Source: https://divisbyzero.files.wordpress.com/2008/09/konigsbergdrawing.jpg (access 18.01.2018)

Fig. 2. **The seven bridges of Konigsberg**

The first problem was the question posed by Leonard Euler: "Is it possible to go from any part of the city Konigsberg (now Kaliningrad) through each of the bridges exactly once, and return to the starting point, without flowing the river? " The Euler's answer was: "no". Leonhard Euler (born 15.04.1707 r. in Basel, Switzerland, died 18.09.1783 in St. Petersburg, Russia) was a mathematician, physicist and astronomer, one of the founders of modern mathematics including graph theory. When studying this problem, Euler came to the ingenious idea to mark separate parts of the city with vertices and the bridges as the connections between them. That was how he constructed a graph with four vertices and seven connections, as we can see on the Figure 2. Solution to the problem of the seven bridges is regarded as the first theorem of graph theory.

There exists a similar problem discovered in early 1960s by the Chinese mathematician, Kwan Mei-Ko, and after him is called Chinese Postman Problem. The problem that the Chinese Postman faces is: he wants to travel along every road in a city in order to deliver letters, with the least possible distance. The question is how to find a shortest closed walk of the graph in which each edge is passing at least once, rather than exactly once. The solution in graph theory is an Euler cycle in a connected, weighted graph and is called the Chinese Postman problem, also called Postman Tour or Route Inspection Problem (Kramberger, T., Zerovnik, J., 2007).

Practical applications and solutions

Graph theory as a part of mathematics concerned with the study of their properties is an important mathematical tool used in many different areas, such as computer science, calculus, economics, management, sociology or logistic in business (Rungis Ruohonen K.,2013). Creating algorithms for designating certain properties of graphs is one of the important areas for action. These algorithms are used to solve many practical tasks. Graph theory is also present in widely understood economic applications. A wide field for applications of graph theory is, among others, logistics in agribusiness. In the area of logistic applications, the analysis of goods' flows in supply chains can be distinguished (Scholz-Reiter et al., 2011).

A common practical application in this field is also the search for the shortest possible route between wholesale markets, stores, agricultural fields and others places connected by roads. Data structures based on graphs are also used in the analysis of the network of dependencies of entities operating on various types of markets. In this approach, the axis of the issue is to identify interactions between individual market participants, which should lead to a better understanding of

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the specificity and functioning of the analysed market segment (Jackson, 2010). Presentation of the flow of information between specific organizational units may allow for the disclosure of an informally existing organizational structure, the role of which may be more significant in relation to the official structure (Cordella A., 2006).

The management seems to be an interesting analysis of the interaction in the company's organizational structure (Konig, Battiston, 2009).

Graph as a set of points (called vertices or nodes) connected by edges (or arcs) can be a representation of cities, villages, wholesale markets, stores, agricultural fields and others places connected by roads. It is necessary to use graphs with specific properties, e.g. those in which the route passing through each edge or by each vertex exactly once, and ending at the point of exit, there are called graphs Euler's and Hamilton's. The structure of the graph we can provide by means of certain matrices associated with the graph. We distinguish between the so-called matrix connections and incidence matrix.

If the graph contains n edges, matrix (neighbourhood) is the matrix of square n × n and contains the elements 0 and 1:

- 1-when there is an edge (i, j), and
- 0-when the edge (i, j) is not present.

The authors have decided to present in the paper two real practical cases of application the graph theory in logistic of agricultural sector:

1) choice of the shortest way of delivery agricultural products from the Wholesale Market to some shops in the cities;

2) county snow removal plan X-on the rural areas, what is connected with collection the milk from contracted farms.

The Figure 3 shows the first case with the task to find the shortest way between certain cities. There is a special task for salesmen delivering, for example agricultural products from certain place. This case was focused on the delivering of some fruits and vegetables from Agri-food wholesale market "Targpiast" in Wroclaw. Cross-regional, agro-food wholesale market "Targpiast" was founded in 1990. Its basic function is to support businesses in Wroclaw, Lower Silesia and the neighbouring regions in terms of their supply of food - especially fresh vegetables and fruits. The Wholesale market supplies food, fruit and vegetable stores, retail chains, local bazaars and marketplaces, catering companies, restaurants and other catering facilities. The location of this wholesale market "Targpiast" is near the downtown of Wroclaw. City Wroclaw bypasses facilitate the accessibility of the market, without disturbing the communication rhythm of the city.

In the Figure 3, each pair of cities is connected by edges, so one can say that this is the full Euler's graph. The cities were assigned individual vertices, then the authors used description: 1. Wroclaw, 2. Walbrzych, 3. Legnica, 4. Poznan, 5. Jelenia Gora.

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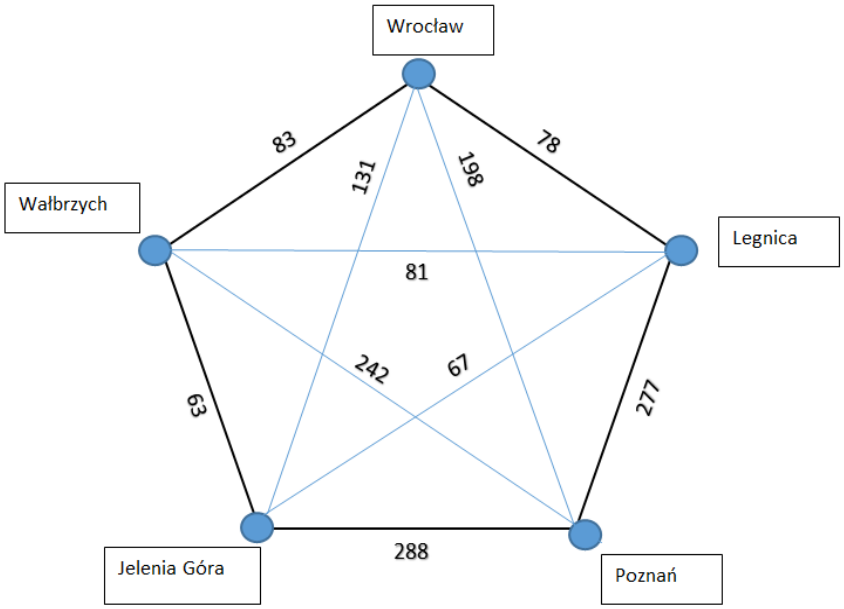


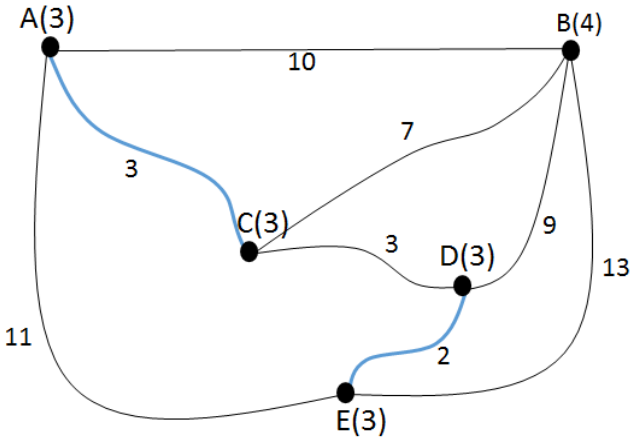
Source: authors' calculation

Fig. 3. **Graf-weight (weight = distance in km) in which the vertices are cities**

By carrying out the steps of the algorithm, on the base of a matrix describing the graph, we received: such order [4.1] [2.5] [3.4] [1.2] [5.3], and after arrangement: [1.2] [2.5] [5,.3] [3,.4] [4.1].

That is: Wroclaw – Walbrzych, Walbrzych – Jelenia Gora, Jelenia Gora – Legnica, Legnica – Poznan, Poznan – Wroclaw. The shortest route is 598 km. A traveling salesman has to beat the following route: from Wroclaw via – Walbrzych – Jelenia Gora – Legnica – Poznan – and back to Wroclaw.

The second case is: how to minimize the number of kilometres of snow blower's travel using the principle of "Chinese Postman". On the next graph we observe next real problem: county's snow removal plan in the rural areas, which is crucial also for collection of milk from farms.



Source: authors' calculation

Fig. 4. **Graph of the snow removal plan**

According the procedure, it is necessary to make the next steps for finding solution.

Step 1: It is necessary to verify whether the graph is consistent, that is, if for every pair of vertices there is a path that connects them. The above graph is consistent.

Step 2: We are searching for in the graph all vertices of odd degree. These vertices are: A, C, D, E.

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Step 3: Assign pairs of vertices. AC, CD, DE, AE, and AD and CE are added (they do not exist on the graph.

Step 4: Create combinations of vertices and connections we set the shortest path connecting all found the vertices at odd degrees. AC, DE 3,3 = 6 . Select these edges, because they have the smallest sum of edge weights: AE, CD;11,3 = 14 and AD, EC; 6, 5 = 11.

Step 5: The edges of the marked on the graph in blue „double" in the graph, it will be the return path.

Step 6: It is necessary to go throw graph: A E B A C B D E D C A.

Step 7: We add up the length of roads:

11 + 13 + 10 + 3 + 7 + 9 + 2 + 2 + 3 + 3 = 63 km.

Solutions obtained through the graph theory are very helpful to the organisers of different events, actions connected with routes, for those who are resolving logistical problems also for agriculture sector.

Conclusions

Rationalization of logistic processes can be one of the ways to increase the competitiveness and efficiency of agribusiness enterprises. It results in the appropriate organization of supply and distribution, taking into account the preservation of the appropriate quality of fresh products and effective inventory management. In the same time, with the growing scale of agribusiness enterprises, the role of logistics management in purchasing raw materials, their quality and the flow of information in the supply chain of fresh products is growing.

One of the instruments which can help to rationalize logistic process in agricultural sector is application of theory of graphs. In the paper, authors have described the connection of real-life logistics problems and graph theory. One can see the heterogeneity between the systems of organisation but furthermore the complexity of finding a simple solution information for a stakeholder.

Graphs and networks as decision support tools are a graphical model reflecting the decision problem with its structure, quantifiable and enabling to obtain an accurate mathematical solution in the form of a precise assessment. One of practical cases of an application of the Chinese Postman Problem is planning of milk collection in the rural areas routing. In order to save the cost on the fuel for milk tank, company can model the stops the vertex and the road as the edge in the tank route, then using the graph theory to obtain the optimal route that can meet the target group using the minimum of fuel but crossing every road once.

There are many practical real-life logistics problems, for example: milk delivery, agricultural products delivery to wholesale markets, school bus routing in the rural areas and many others. Optimizing the time and costs of delivering of agricultural products with graph theory is an important element of logistics management.

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