Adolescents' Mathematical Competence Formation Influencing Factors

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Abstract: Mathematical competence has been identified worldwide as one of the key competences for personal fulfilment, social and economic inclusion in the knowledge society of the 21st century. Most European countries, including Latvia, are transforming their education systems, so it is very important to have a valid understanding of adolescents' mathematical competence, yet it is not defined. Some of the discovered factors that influence this competence are currently neglected in terms of education reforms, so this article is timely, urging to think over the essential accents of the change. The article also questions the previous experience of transforming mathematics teaching – reviewing current national policies for developing the key competences for lifelong learning. Many countries, including Latvia, have come across a disproportion of substantial investments into teaching mathematics and, despite this, low pupil performance. The aim of the article is to determine factors of the formation process of the pupils' mathematical competence. To explore it, during the study the most popular approaches on defining mathematical competence have been compared. These approaches have many differences, but also five closely related factors. Firstly, it is the way of teaching and learning. Some researchers believe, the style of teachers' work, views and personality traits are most crucial. Secondly, learning materials, including textbooks and online resources encouraging the pupils to work more independently. The third factor to be discussed in the article is the pupils' individual characteristics: motivation, attitude, persistence, values. Speaking of persistence, it is believed to be more important than being talented. Next factor is the learning environment, both at school and beyond it: at home, on a trip, while shopping. Despite the fact that mathematical competence is not normally measured in these everyday situations, it is gained through these practical activities. Finally, the society brings many stereotypes and influences views on mathematics as too hard or boring to go deep in it. All these (and other) factors can be turned to support formation of mathematical competence, instead of blocking it. The article will reveal how these factors are linked and how they are helpful.

Keywords: mathematical competence, formation, model, school education, 8th graders.

Introduction

The role of schools in the modern sense is to prepare pupils for successful fulfilment of their skills in the fast changing life and to gain willingness to continue learning throughout their lives. Today's pupils' success in their future careers largely depends on their knowledge, skills, abilities, motivation and other factors – in one word it can be called competence.

United Nations Educational, Scientific and Cultural Organization (UNESCO) named mathematical competence as one of the most crucial competencies. It is also identified as one of the key competences by the European Union with an aim to decrease the share of 15-year-olds with insufficient abilities in reading, mathematics and science below 15 % by 2020. (Strategic Framework…, 2009). In 2015 the share of low achievers in mathematics (below level 1 and level 1 combined) was 22,1 % (it was the same also in 2012), but it differs dramatically around the world (Eurostat, 2016). For instance, in Estonia only 11,2 % of pupils are low achievers in math, in Latvia – 21,5 %, but the worst figure in Eurasia is in Turkey – 51,3 %; across the world it is in Mexico – 56,6 % (Table 1).

Researchers are concerned not only about the lack of knowledge, but also about the breakdown of results – it is far from the normal distribution. Approximately one fifth of Latvian 8th graders could not solve even the easiest mathematical tasks in the Programme for International Student Assessment (PISA) test. Meanwhile, the share of the highest performing pupils in mathematics is only 0,7 % (it was 1,5 % in 2012), when the average in the Organisation for Economic Co-operation and Development (OECD) countries is more than twice higher – 2,3 % (Geske, Grīnfelds, 2016).

To check if the data from PISA is correct, in 2014 Latvia organized test on some components of mathematical competence such as outer communication, which includes ability to express ideas, using accurate mathematical terminology, critical thinking, modelling and problem solving. The statistics were quite shocking – 90,9 % of pupils could do the task, but only 40,4 % could explain what they are

doing in writing. Pupils know the algorithm, but do not know, why it works. Pupils also did not even try to solve tasks, where they are supposed to model tasks with practical content and which does not have one correct answer (Vilciņš, 2015).

Table 1

Percentage of students below Level 1, at Level 1 and at Level 6 in mathematics (OECD, 2016)

Country	Below Level 1	Level 1	Level 6
Estonia	2.2	9.0	2.9
Japan	2.9	7.8	
Denmark	3.1	10.5	5.3
Finland	3.6	10.0	2.2
Germany	5.1	12.1	2.9
Korea	5.4	10.0	6.6
Latvia	5.7	15.8	0.6
United States	10.6	18.8	0.9
Turkey	22.9	28.4	0.1
Mexico	25.5	31.1	0.0

Mostly due to these numbers, over the last decade mathematical competence has become one of the most important educational priorities at the European policy level. In recent years, but especially since 2007, an overwhelming majority of European countries have revised mathematics curriculum with the introduction of results-based approach, where the focus is on pupils' competencies and skills rather than on theoretical knowledge. Latvia is also approaching to implement competence based education starting with 2018, foremost in primary schools and then in 10 years leading to the high school.

Additionally, the issue of mathematical competence is now in the spotlight thanks to ongoing secondary school curriculum reform. The main difference of targeted competence based education versus the current content is that the pupil should be able not only to apply the given formulas for a specific task, but also to recognize and define the problem, choose the appropriate method to solve the problem, to use a method in a new unfamiliar situation, assess the significance of solving the problem. To fulfil this aim schools are about to switch from subjects oriented content to transversal competences, while subject is only a way to develop different competencies (Oliņa, 2015).

The aim of the article approach to determine factors of the formation process of the pupils' mathematical competence. In terms of reforming Latvian education, it is particularly important to summarize recent theoretical findings on mathematical competences, to highlight factors that influence mathematical competence and show their interdependencies.

Methodology

To identify the factors affecting mathematical, the history of the term "competence" in global and Latvian literature has been explored. After some debate on different explanations of this term the definition by Tatjana Koķe has been chosen as the basic: competence is the ability to choose the most appropriate tools to the situation or the operation, based on the knowledge, and to adequately apply them (Koķe, 2003). This definition was formulated for a research on continuous education. It is much broader and more general then other explanations, where competence includes knowledge, skills and attitudes that qualify to do some work or level of task.

Most definitions of mathematical competence (Niss, 2011; Turner, 2011; Laursen, 2010; Casserly, 2012; Balčiūnas, Macaitienė, 2013) have some basic things in common, for instance, almost every definition mentions such skills like communication, problem solving, modelling and reasoning, while there are still some aspects, that are unique – personality traits like persistence and curiosity are cited only in a few formulations of mathematical competence.

Usually mathematical competence is explained through listing certain skills and behaviour. Almost every definition formulated in this decade includes a reference on using mathematical skills in diverse

contexts. Trying to ease and speed up processing of data some authors use interpretation of mathematical competence as an average ability to solve different tasks, which is usually converted into points or percentage. Finally, other researchers highlight normally three or four mathematical content fields and make conclusions about mathematical competence of a pupil based on results in chosen topics.

Based on the exploration of evolution of the understanding of the term "competence" and different attempts to define mathematical competence in different contexts, the definition of adolescents' mathematical competence was composed:

Adolescents' mathematical competence is the ability to initiate, responsibility and self-confidence to perceive, understand and address the diverse situations, to operate in quantitative and generalized patterns; critically evaluate and justify the results, which are able to anticipate result before a solution.

To have a common sense about these definitions is especially crucial now, when Latvia is in the way to undergo a huge, ambitious reform of school compulsory content. It has to become more competence based, involve pupils in more active ways of learning, give them the opportunity to gain their own knowledge. It emphasizes from the teacher in the centre of the learning process to pupil as the main figure in learning.

The main idea of the reform is to motivate teachers to cooperate more and to review subjects' curriculum, leaving only the content that is relevant and meets today's needs. This project is in its early stages, so it is timely to discuss, if these two factors – cooperation of teachers and reviewing the content – do affect mathematical competence the most and which closely related factors are out of the focus, but should be at least discussed, if not included in the final model of competence based education in Latvia.

Previous major project in mathematics education shows, that these two factors alone are not enough. 50 schools in Latvia since 2008 have participated in a pilot project on improving the quality of teaching science, mathematics and technology in secondary schools. Project was held by the Science and Mathematics Education Centre of the University of Latvia (DZM). While working on improvement of mathematics teachers' teaching skills and popularizing the idea of pupil centred learning, from 2008 to 2015 DZM experts observed 205 mathematics and science lessons.

The results are frustrating. Observers found out that even teachers, who are in this project and receive guidance on competence centred education, reproductive and superficial learning process still dominated. During the lessons pupils had limited options of higher cognitive level activity. Lessons are mainly (52 %) frontal, they mostly give pupils "ready knowledge" (Namsone, 2015). This shows that even accordingly trained teachers theoretically agree with the centuries-old idea of reform pedagogy that teachers should not give anything ready to their pupils but motivate them to observe, analyse, compare, combine, model, make concepts, but teachers usually still do not follow these recommendations.

DZM project also concentrated on communication as a crucial skill for both teachers and pupils. According to the project design, teachers were about to learn to express their thoughts precisely and in short, while pupil – to explain their ideas and results, using appropriate terminology. Despite this, research shows that conclusions are still made by the teachers, pupils do laboratory tests following particular steps. If pupils are struggling with laboratory tests, teachers usually do the step instead of them or give direct instruction instead of asking questions that could help the pupils to come up with the correct idea.

In short, this pilot project shows that improving content and teaching teachers some theories and technics of teaching does not improve the overall experience in education, because the teachers do not change their style of teaching. Thanks to this project, teachers cooperated in small groups, discussing the aim of the lesson, its strong aspects and needs of improvement, but improving effect on lessons of these activities was disproportional to the time spent, as teachers did not feel free to express their criticism so everyone got a mostly doubtful or sometimes even wrong impression that they are doing well: believe that their way of teaching is appropriate for teaching scientific inquiry while in fact lesson focuses on applying what has been learned through a sample.

Results and discussion

Using the definition of adolescents' mathematical competence and findings of respectable mathematician methodologists (Колмогоров, 1988; Пуанкаре, 1990; Dieudonné, 1975), the model of mathematical competence crystallized, showing factors that influence mathematical competence (Figure 2).

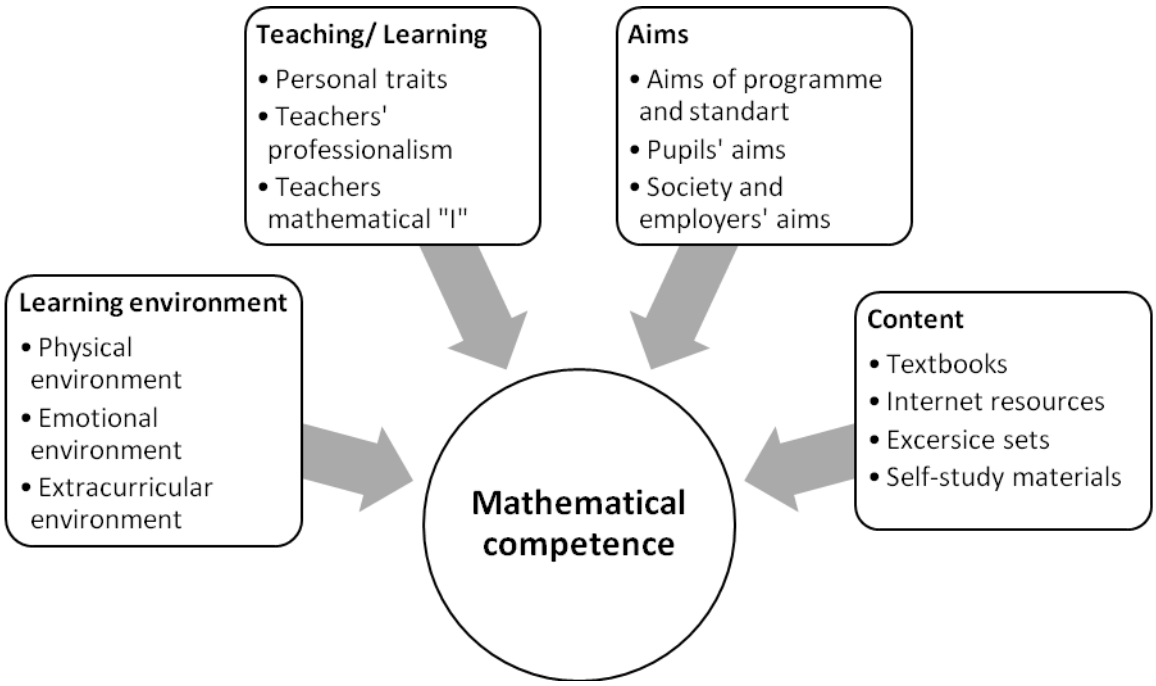

Figure 2. Factors influencing mathematical competence.

The analysis of literature shows that besides the teaching style, cooperation and relevant content, there are many other factors influencing learning in general and gaining mathematical competence in particular. Most authors are concerned that formation of mathematical competence depends on the **learning environment** (Fernāte, 2011; Geske, Grīnfelds, 2016; Wood, 1998; Kahneman, 2011; Hattie, 2012; Eurydice, 2011; Cunska, 2013). The involvement of physical environment into teaching mathematics gives a platform for more stable knowledge. It also leads to higher motivation, for example, if the posters of pupils are used to give more mathematical sense to walls of a classroom, pupils do not only learn necessary facts through the physical environment, but also improve their emotional environment – celebrating small victories this way gives pupils a conviction that they can be successful in mathematics. Lack of self-confidence is a very strong obstacle in learning mathematics. As a part of the learning environment extracurricular environment should also be mentioned. Friends, groups of interests and sports clubs affect how easily pupils can apply mathematics in many different situations and contexts, because they can judge and model from their own experiences.

The next factor that influences mathematical competence is **teaching and learning**. The discussion at the moment is mainly on teaching – cooperation of teachers, new worksheets, different tests and exams – but the researches show that learning should not be neglected. Many authors (Колмогоров, 1988; Tough, 2012; Mencis, 2010) stress the importance of personal traits of pupils. P. Tough is convinced that persistence and curiosity alone outweigh all other factors of learning. Meanwhile, A. Kolmogorov claimed that mathematics intuition and willingness to learn are more important than, for example, ability to memorize large amounts of information, facts, formulas. There are plenty of researches and recommendations on how teachers should study and teach (Ļubkina, 2007; Lāce, 2010; Hattie, 2012; Upīte, 2013). The problem appears to be the implementation of these ideas, since teachers reject change or lack knowledge to use these methods.

The aims are the third part of mathematical competence model is **aims**. It includes many types of aims that altogether impact the formation of mathematical competence. First, those are aims of the education program and standard. Both are usually general to meet needs of different schools and pupils. Second, aims of the pupil. This includes motivation to learn, vision of applying school curriculum later in

profession or everyday life. Third, the opinion of the society on what should be taught in school. When reforming education system or even a part of it should be kept in mind, that speaking about education society has a tendency to be conservative, to require the same methods and content of learning that adults remember form their experience in school, but which should also prepare pupils for inclusion and effective work in future society and more specific – labour market. If all these and many other aims related to education can be combined, pupils normally gain the highest possible result. Good practice example of aim based education system is, for instance, Norway (Education – from..., 2007).

The Content is the fourth component of this model. Undoubtedly, textbooks and other resources pupils use while learning are extra important (Cunska, 2013; France, 2010; Niss, 2011). Content is one of the ways how to provoke and maintain the interest about learning.

Conclusions

Mathematical competence is undoubtedly one of the crucial competences for persons' fulfilment throughout the life. Despite this, adolescents' mathematical competence is a concept explored by few researchers. Exploring existing definitions of mathematical competence, the model of formation of mathematical competence has been created. Description of the model includes analysis of currently accented components of mathematical competence. Based on researches in pedagogy and other fields, the missing components of this model are identified in context of ongoing secondary school curriculum reform towards competence based education. Summarizing this theoretical research adolescents' mathematical competence has been defined.

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