THE COMPARING MATHCAD AND WOLFRAM ALPHA IN THE PROCESS OF INTEGRATION OF SOME FUNCTIONS

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Abstract: Mathematics Department in the LUA has good experience in integrating IT program MathCad in mathematics study process for bachelor and Matlab for master programmes. The rapid changes in computer technology led to computer algebra systems - besides MathCad and Matlab there are also Wolfram Alpha, Mathematica and Maple which are used for education of future engineers. The paper describes and analyzes commercial software MathCad and Web resource "Wolfram Alpha", which is popular and helpful as free math software. The aim of the research is comparing two math softwares MathCad and Wolfram Alpha as a tool for calculation integration. For receiving results in MathCad in a convenient form, students are required to have theoretical knowledge. It can be a negative aspect for MathCad, but also a positive aspect for the student's knowledge. In Wolfram Alpha students have possibility to see alternative forms of result. Wolfram Alpha gives step-solutions. Although it is enough to use computation ability for most problems we meet in teaching of Math, in certain cases the program function becomes necessary and very helpful. Students can get necessary knowledge using MathCad to start using other more difficult mathematical softwares in future.

Keywords: rational functions, integration, software "MathCad", web resource "Wolfram Alpha".

Introduction

EU directives distinguish 8 key competencies, which should be developed for lifelong learning (Key competences…, 2006). One of them is for mathematical literacy and competences in science and technology. Mathematics knowledge and competences become essential in lifelong learning process. Mathematics is a discipline which is required as a background for specialists who work in environmental protection, engineering, construction, business, telecommunication, textile, new energy sources, etc.

The professors of Institute of Mathematical Sciences have identified eight key mathematics competences (Laursen, 2010). Competences concerning the ability to ask and answer questions about and by means of mathematics are: *Mathematical thinking* (mastering mathematical modes of thought awareness of the types of questions that characterise mathematics, ability to pose such questions**,** insight into the types of answers that can be expected); *Problem handling* (being able to formulate and solve mathematical problems, i.e., put forward different kinds of mathematical problems, pure and applied, open and closed, solve mathematical problems, if already formulated, whether posed by oneself or by others, and, if necessary or desirable, in different ways); *Modelling* (being able to analyse and build mathematical models concerning other areas); *Reasoning* (being able to reason mathematically). Competences *concerning the ability to deal with mathematical language and tools are: Representation* (being able to handle different representations of mathematical entities); *Symbol and formalism* (being able to handle symbols and formal mathematical language); *Communication* (being able to communicate in, with, and about mathematics); *Aids and tools* (being able to make use of and relate to the aids and tools of mathematics (incl. IT)).

The increasing expansion of Information Technologies worldwide creates a new social relationship – the information society. Therefore the necessity of the search and use of information has emerged. The concept of literacy has expanded and acquired a new name – information literacy. The objective of the education system is the development of information skills (Zeidmane, Cernajeva, 2011). An individual, who is information literate, is aware of the necessity of information, he/she has the ability to obtain information, use it effectively as well as estimate the result according to the requirements of the task and the relevant knowledge growth (Pickering, 2004).

As concerns the content of Mathematics Study programs at university, a compromise should be found between the acquisition of the fundamental knowledge, acquisition of know-how application of knowledge as well as the use of IT software in the calculation (Zeidmane, 2012). Mathematics Department in the LUA has good experience in integrating IT program MathCad in mathematics study process for bachelor and Matlab for master programmes. In mathematics study process for bachelor are 0.5 of contact lessons per week as laboratory work (MathCad) in addition to contact lessons per week as lectures and practical works. The teacher of MathCad in cooperation with the teacher of the practical work hand out homework to students about the corresponding topic which they must solve on the paper, demonstrating the process of solution step by step, afterwards in practical classes students check solutions themselves using MathCad programme. The analysis of the students' success and the students' survey at the LUA shows that students prefer the introduction of MathCad in the study subject of mathematics instead of learning MathCad as a separate subject supplied by mathematical examples. Students enjoy comparing the results of their individual tasks with the results obtained via MathCad, that, in turn, increase the motivation to solve more mathematic problems, since students are interested in solving several variants. The final tests complete the study period of individual tasks and MathCad problems thus the final tests show the improved scores.

The rapid changes in computer technology led to computer algebra systems - besides MathCad there are also Wolfram Alpha, Matlab, Mathematica and Maple which are used for education of future engineers. On the other hand, education is no longer limited to classrooms. Internet can play an important role in education. The Internet can be used to refer to information on different subjects, including mathematics. Students can use web resources for understanding the course of mathematics and for getting help, for example, Wolfram Alpha (WolframAlpha web resource, 2012). Further in examples software is discussed with the focus on possibilities, risks and limitations.

The computer and information competence of students of technical disciplines is a component of information literacy (Figure 1).

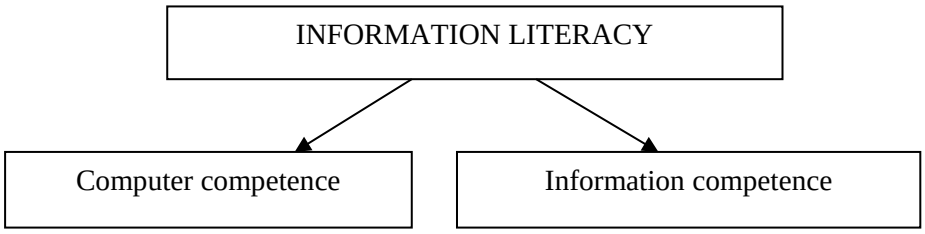

Figure 1. Component of information literacy.

Computer competence includes focusing on new knowledge of computer facilities and modern specialized computer programs. Information competence includes an important skill to estimate information critically and competently (Asherov, Bogdanova, 2007).

Methodology

WoframAlpha introduces a fundamentally new way to get knowledge and answer – not by searching the web, but by doing dynamic computations based on a vast collection of built-in data, algorithms, and methods. (WolframAlpha web resource, 2012)

The section Mathematics contains Elementary Mathematics, Numbers, Plotting&Graphics, Algebra, Calculus and Analysis, Geometry, Number Theory, Discrete Mathematics, Applied Mathematics, Logic & Set Theory, Mathematical Funkctions, Advanced Mathematics, Mathematical Definitions, Famous Math Problems. In this paper we consider the subsection Calculus & Analysis. This subsection consist of Integrals, Derivatives, Limits, Sequences, Sums, Products, Series Expansions, Coordinate Geometry, Differential Equations, Complex Analysis, Vector Analysis, Integral Transforms, Numerical Analysis. Indefinite Integrals, definite integrals, multiple integrals, numerical integration and integral representations are considered in the sub-subsection Integrals. In this paper only indefinite integrals are discussed. The knowledge about indefinite integrals is applied in definite integration, double integrals, ordinary and partial differential equations. To symbolically evaluate

indefinite integrals, you can write "int *expression* dx" or "integrate *expression* dx" and press "enter" to get the result of integration and other possibilities.

MathCad is industrial standard technical calculation tool for engineers worldwide. MathCad delivers all the solving capabilities, functionality, and robustness needed for calculation, data manipulation, and engineering design work. By combining calculations, graphs, text, and images in one document, MathCad enables knowledge capture and publication that aid management of large projects. MathCad provides online Tutorials; QuickSheets of working examples of MathCad functions and applications; and Reference Tables with math, science, and engineering formulas. In this paper we use the MathCad 14 version. MathCad lets enter equations, text, and plots anywhere in the worksheet. Each equation, piece of text, or other element is a region. By defining variables and functions, students can link equations together and use intermediate results in further calculations.

When students evaluate an expression numerically, MathCad returns one or more numbers. When MathCad calculates symbolically, the result of evaluating an expression is generally another expression. Students can make the numeric and symbolic processors work together, so that an expression is simplified before the numeric processor calculates it. To symbolically evaluate indefinite integrals, you can use MathCad's indefinite integral operator. Then the placeholder for the integrand is filled in. The integration variable in the placeholder is placed next to the "d". It is necessary to click "$\rightarrow$" on the Symbolic toolbar and press "Enter". The Symbolic menu commands can be used. The Symbolic menu commands perform the same manipulations as many of the keywords listed in online Help. For integration it can be applied successfully the commands: simplify, substitute, factor, parfrac.

Integration of rational functions. One of the most important classes of elementary functions, whose integrals can be found in a comparatively simple way, and always means elementary functions, is rational functions. Except for a few very special cases, currently we have no way to find the integral of a general rational function.

A rational function is a function which is the quotient of two polynomials. Consider integrals of the type $\int \frac{P(x)}{x^2+px+q}dx$ where $P(x)$ is a polynomial, $p,q \in R$. If the degree of the polynomial $P(x)$ is greater than 1, the division of $P(x)$ by x^2+px+q results in a polynomial $Q(x)$ and a polynomial $ax+b$, as the remainder. Consequently $\frac{P(x)}{x^2+px+q} = Q(x) + \frac{ax+b}{x^2+px+q}$. The integration of the polynomial $Q(x)$ does not present any difficulties and hence the problem reduces to integrating a fraction $\frac{ax+b}{x^2+px+q}$, if $a^2+b^2 \neq 0$. If $x^2+px+q = (x-x_1)(x-x_2)$, where x_1 and x_2 are two different real numbers, then there exist real constants *A* and *B* such that $\frac{ax+b}{x^2+px+q} = \frac{A}{x-x_1} + \frac{B}{x-x_2}$, what is useful for integration. Unknown constants *A* and *B* are found by "the method of indefinite coefficients". If the polynomial $x^2+px+q = (x-x_1)(x-x_2)$, where x_1 and x_2 are two different real numbers, then there exist real constants *A* and *B* such that $\frac{ax+b}{x^2+px+q} = \frac{A}{x-x_1} + \frac{B}{x-x_2}$, what is useful for integration. It is important for the student to know about integration methods "the method of indefinite coefficients".

Results and discussion

Consider the case when the polynomial x^2+px+q has a double root or is positive on an interval. Using software "MathCad" for calculating this examples, students have no problem with solutions. The result is given in easy to use form. Students should remember that the computer algorithm might simplify expressions different from what we are used to. Also, it does not give the integration constant "$+C$".

The software MathCad is commercial software, therefore students can also use free web resource Wolfram Alpha. Using Wolfram Alpha, students can have a problem with the writing expressions. MathCad lets students enter the text and mathematical expressions anywhere in the worksheet and type equations as students are used to seeing them, expanded fully on computer screen. Wolfram Alpha lets students enter the text in a certain place using specific symbols as brackets, special functions for roots and other. A positive aspect in Wolfram Alpha is the possibility to get more

information about functions in results. Students can use what kind of information (documentation, properties or definition) they want to see the function in the result. Students can get the definition and properties of these functions visiting the website (WolframMathWorld web resource, 2012).

The student can use Wolfram Alpha not only for getting integration results. It is important that they can see steps of integration. It is a good possibility to learn the methods of integration or to understand what kind of method can be used for this example. The negative aspect is that students can use this possibility only for three integrals during a day. If they want to work more, it is the service for a fee. Students get the alternate form of the result among the results in Wolfram Alpha if it is possible.

Example. Calculate the indefinite integral $\int \frac{dx}{x^2-3x+2}$.

The polynomial $x^2 - 3x + 2$ has two different roots $x_1 = 1$, $x_2 = 2$ (both the real numbers). The result in MathCad is given as in Figure 2.

$$\int \frac{1}{x^2 - 3\cdot x + 2}\, dx \rightarrow -2\cdot \operatorname{atanh}(2\cdot x - 3)$$

Figure 2. The result of integration for function $\frac{1}{x^2-3x+2}$.

The integration result has inverse hyperbolic tangent. This result is difficult for understanding for students. Almost all students do not know about the inverse hyperbolic tangent.

This rational function $\frac{1}{x^2-3x+2}$ can be written in the form of the partial fractions decomposition and then integrate new expressions (1).

$$\int \frac{dx}{x^2 - 3x + 2} = \int \left(\frac{1}{x-2} - \frac{1}{x-1}\right) dx = \ln|x-2| - \ln|x-1| + C \quad (1)$$

The result is easier to understand for students and if it is necessary to continue work with multiple integrals or differential equations. The similar result is obtained from MathCad result, using the formula (2) and (3)

$$\operatorname{arctanh} x = \frac{1}{2}\ln\left|\frac{x+1}{x-1}\right|, \quad (2)$$

$$-2\operatorname{arctanh}|2x-3| = -\ln\left|\frac{2x-2}{2x-4}\right| = -\ln\left|\frac{x-1}{x-2}\right| = \ln|x-2| - \ln|x-1| \quad (3)$$

These transformations require additional knowledge for students.

Sometimes students can get result more easily using MathCad functions (simplify, expand, factor, parfrac etc.) (Figure 3).

$$\int \frac{1}{x^2 - 3\cdot x + 2}\, dx \rightarrow -2\cdot \operatorname{atanh}(2\cdot x - 3) \text{ simplify } \rightarrow \ln(4 - 2\cdot x) - \ln(2\cdot x - 2)$$

Figure 3. The result of integration for function $\frac{1}{x^2-3x+2}$ using command “simplify”.

As regards MathCad, students can at first find partial fractions decomposition for a given rational function (the MathCad function parfrac) and then evaluate indefinite integrals (Figure 4).

$$f(x) := \frac{1}{x^2 - 3x + 2} \text{ parfrac } \rightarrow \frac{1}{x-2} - \frac{1}{x-1}$$

$$\int f(x)\,dx \rightarrow \ln(x-2) - \ln(x-1)$$

Figure 4. The result of integration for function $\frac{1}{x^2-3x+2}$ using at first command "parfrac".

Students get a result in a way that is easy and understandable for them, using Wolfram Alpha without transformations (Figure 5).

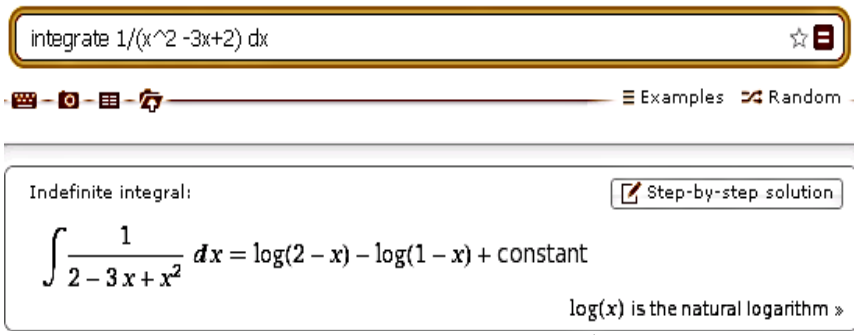

Figure 5. The result of integration for function $\frac{1}{x^2-3x+2}$ in WolframAlpha.

Choosing step-by-step solution, students can see only one method of integration (Figure 6, Figure 7).

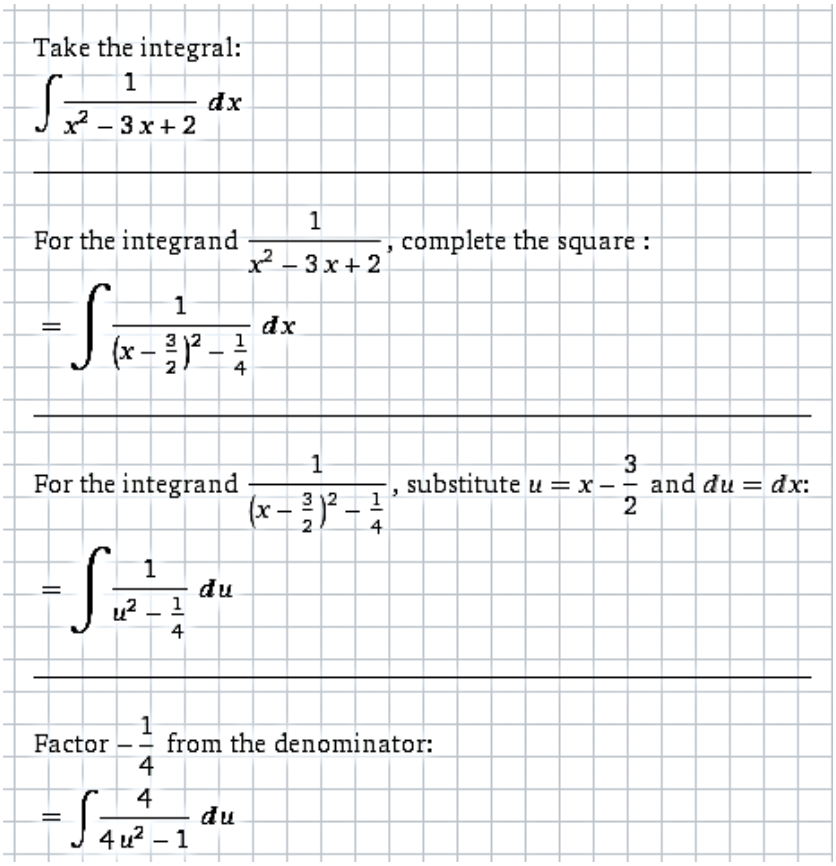

Figure 6. The integration using step-by-step solution in WolframAlpha (the beginning of the integration).

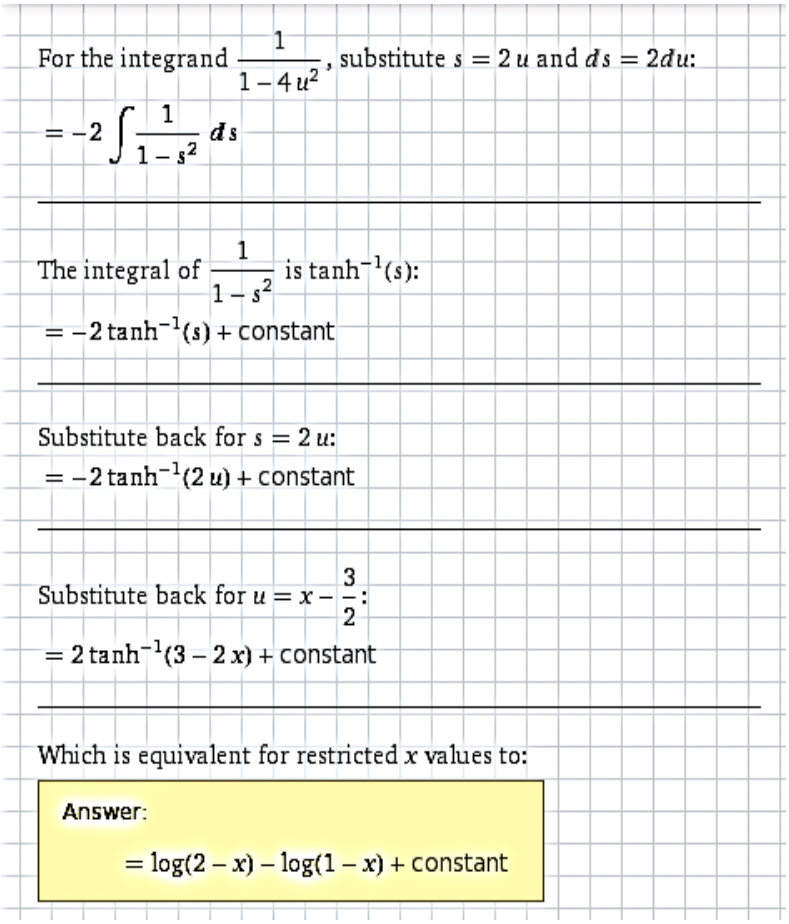

Figure 7. The integration using step-by-step solution in WolframAlpha (the end of the integration).

There is a new function - inverse hyperbolic tangent in the process of integration. It is not understandable for students how the inverse hyperbolic tangent can be used to get the function of the natural logarithm.

It is necessary to calculate the indefinite integral $\int \frac{11x^3-107x+108}{x^4+x^3-30x^2+76x-56}\,dx$. If students integrate rational function as given in MathCad, it takes more time to get the result. The similar result can be obtained faster if at first students use the method of partial fractions and then integrate these fractions.

Similar results are obtained by both programmes - the web source "Wolfram Alpha" and MathCad. Choosing step-by-step solution, students can see the method of indefinite coefficients using partial fractions. Also they can see the alternative forms of result.

Conclusions

1. Software "MathCad" is commercial software. Web resource "Wolfram Alpha" is free internet resource but with restrictions on the use of additional function (step-by-step solution).
2. With MathCad students have the ability to save the obtained result and it can be used in future. In Wolfram Alpha students have the opportunity to save history and also later get information about examples and about the date when it was done. If they do not choose to save history, students see the result only once and later they have to write it as a new expression.
3. As a result, only one form is given in MathCad. Students can get special functions. For receiving results in MathCad in a convenient form, students are required to have theoretical knowledge about integration of some rational functions. It can be a negative aspect for MathCad, but also a positive aspect for the student's knowledge. In Wolfram Alpha one form of result is given. On the other hand, students have possibility to see alternative forms of result (if these forms exist).

4. Wolfram Alpha gives step-solutions. It is important for students who want to get more knowledge about integration and different transformations. MathCad has not such function.
5. If some functions are not known, the students can choose reference to Wolfram Mathematica to see definitions and properties of these functions.
6. In MathCad students should know methods of integration to get the results with more functions. In Wolfram Alpha it is possible to work without knowledge of methods of integration.
7. As regards MathCad, it is necessary to have time for explanation and learning how to work in MathCad window. On the other hand, students can get necessary knowledge using MathCad to start using other more difficult mathematical softwares "Matlab", "Mathematica" and others in future. Similar examples can be used in Wolfram Alpha.

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