STRUCTURAL ENGINEERING

ANALYSIS OF BASE FIXED STEEL FRAME BY PLASTIC METHOD

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ABSTRACT

By using an elastic design it is possible to analyse several combinations of loading conditions for which the structure must be designed. However, the rules of superposition are not valid in plastic design. Each possible loading combination has a different failure mechanism and each combination has to be considered separately. The location of plastic hinges in the frame varies with the type of loading, shape and physical properties of the frame. To determine the location and minimum number of plastic hinges needed for a mechanism with the given loading, various analytical procedures may be applied to the frames. The most widely used is the energy and equilibrium method. The main task of this study is to discover which mechanism requires the least load, establishing the limiting capacity of the frame. According to the analysis performed it was determined that the real ultimate load carrying capacity of a frame by the combined or beam mechanism method can be fixed. The frame structure designed by using the ultimate strength analysis resulted in weight savings of the steel framework from 4 to 10 %

Key words: Bending moment, equilibrium method, mechanism method, plastic deformations, ultimate load

INTRODUCTION

Frames can be made of rolled shapes or built-up members, with welded, bolted, or riveted connections. With careful design, attractive and economical structures may be obtained for spans varying from 12 to 60 m. In some instances rigid-frame construction may require a slightly greater amount of steel than a truss-column frame, but the simplicity and speed of erection usually result in appreciable savings (Bresler et al., 1960). Also, the use of welding and the plastic method of design may achieve further savings, so that the use of a rigid base fixed frame becomes economically advantageous.

For a rigid frame, with known loads and support conditions, determination of reactions, internal forces, and bending moments is a statically indeterminate problem. The solution to this problem requires consideration of the stress-deformation relations of the frame components. If the stress-deformation relations are linear, the internal forces and bending moments can be determined by using methods based on the theory of elasticity.

Elastic solution is based on the conditions of continuity and equilibrium, and the assumption that $M_{\max} < M_y$, where bending moment M_y corresponds to the yield stress f_y. If plastic deformation takes place, i.e. local deformation increase without an increase in local stress, forces and moments can be determined by using methods based on the theory of plasticity. Plastic solutions are based on the conditions of local plastification, forming of a collapse mechanism, and conditions of equilibrium. When full plastification occurs at certain critical sections of a frame, it leads to the development of plastic hinges, i.e. when $M_{\max} = M_p$ at these sections. The plastic moment M_p corresponds to the moment at which the deformation begins to increase rapidly.

The ultimate load is usually defined as the load which produces a sufficient number of plastic hinges to convert the structure into a mechanism allowing instantaneous hinge rotation without developing an increased resistance. In order to determine the location and minimum number of plastic hinges needed for the mechanism with the given loading, various analytical procedures may be applied to frames. The most widely used is the energy or mechanism method and equilibrium method.

Many investigations of statically determinate and indeterminate steel frames have been performed and have solved different problems. The pin-supported statically indeterminate frames of the first degree have been studied in textbooks (McCormac, 1992; Salmon and Johnson, 1990; Crawley and Dillon, 1977).

Some approaches for steel frame analysis have been published in journal articles: numerical method in elastic and large deflection analysis of steel frame with non-linear flexible joint connections (Chiorean, 2009), an approach for the modelling of joint flexibility in the nonlinear analysis of steel moment frames (Hjelmstad and Haikal, 2006), and the effects of connection flexibility and material yielding on the behaviour of plane steel frame subjected to static (monotonic) loads (Sekulović and Nefovska-Danilović 2004). This paper aims to develop a simplified method for strength analysis of the base fixed rigid portal frame and to clarify which mechanism requires the least load establishing the limiting capacity of the frame.

DESCRIPTION OF THE METHODS

Mechanism method

The mechanism (energy) method involves an energy theory where each plastic hinge is assumed to have a virtual rotation such that the total internal work can be equated to external work. The external work is represented by the displacement of the supported loads. Let us examine the method in reference to a simple base fixed rigid frame. The frame is assumed to have just reached a mechanism state and each plastic hinge, developing a plastic moment M_p, goes through a rotation θ so small that it is referred to as virtual. The internal work at each plastic hinge can be represented as the product of M_p and θ. The sum of all the work at each plastic hinge, required for mechanism, represents the total internal work W_i. The external work W_e is represented by the sum of the products of loads and their displacements. According to the law of conservation of energy it is possible to permit equating of the external work to the internal one.

The considered base fixed rigid frame, bent with a single concentrated vertical (gravity) load $F_v = c_g F$ at mid span of the girder and equivalent horizontal load $F_h = c_w F$ (Fig. 1a), where coefficients c_g and c_w can be selected depending on the loading variant and fixed load F. Since the shear is uniform and the bending moment varies as a straight inclined line, the plastic hinges can form only in cross sections 1-5. In order to determine the bending moments by using the given loads, let us present the frame by an equivalent system (Fig. 1b). For such purpose the frame is divided into - two half parts and the interaction of both halves is replaced by shear Q, axial force N, and bending moment M_3.

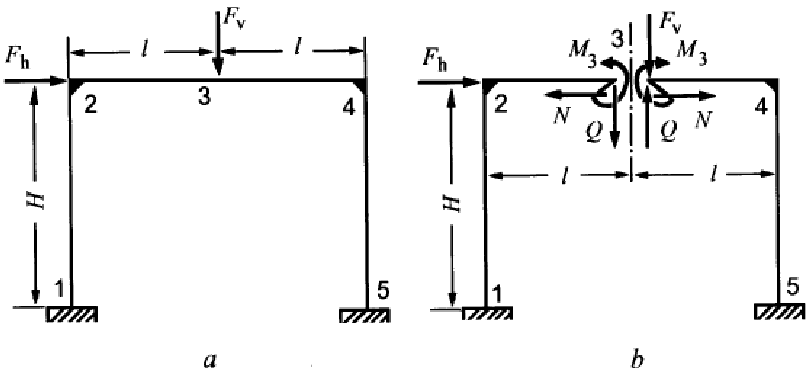

Figure 1. Base fixed frame (*a*) and equivalent analysis scheme (*b*)

Considering the loaded frame shown in Fig. 1, three separate analyses are required: beam-type, panel-type, and the combined mechanism. The main task is to discover which mechanism requires the least load, thereby establishing the limiting capacity of the frame, i.e., its ultimate strength. Because the frame has six reaction components and is thus indeterminate to the third degree, it requires four plastic hinges to create a mechanism. However, in some cases it can require only three plastic hinges for the mechanism.

The beam-type mechanism requires three plastic hinges in cross sections 2, 3, and 4. The kinematically possible state is shown in Fig. 2a. Involving the principle of virtual work and equating external work to internal work $W_e = W_i$ we can get the expression

$$c_w c_g F\,l\,\theta - 4M_p\theta = 0,$$

from which follows the formula for the ultimate load F_u in the case of the beam-type mechanism

$$F_u = \frac{4M_p}{c_w c_g l}\,. \tag{1}$$

The panel-type mechanism requires four plastic hinges in cross sections 1, 2, 4, and 5. Virtual displacements of the frame at ultimate load are shown in Fig. 2b. Taking into account that the displacement of the member 2-3-4 in the vertical direction is negligible, the expression of the full energy of system is

$$kc_g F\,l\,\theta - 4M_p\theta = 0,$$

where $k = H/2l$.

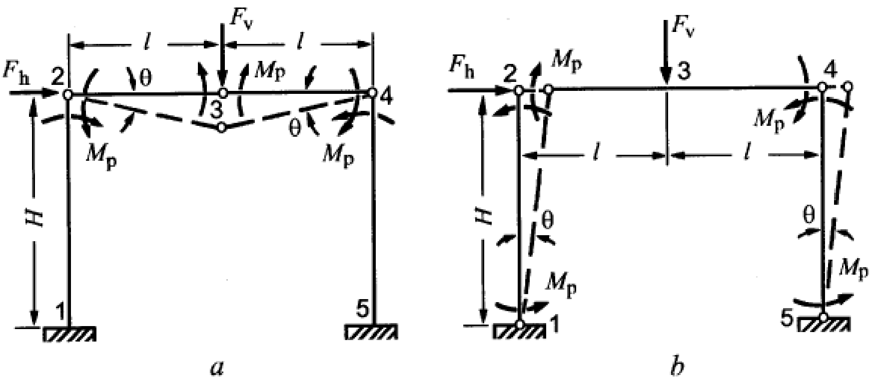

Figure 2. Mechanism method analysis: beam-type (*a*) and panel-type (*b*)

The ultimate load for panel-type mechanism is

$$F_u = \frac{4M_p}{kc_g l}. \quad (2)$$

Assuming that the plastic hinges form in cross sections 1, 3, 4, and 5 (Fig. 3), the ultimate load can be determined according to the combined mechanism. On the basis of the principle of virtual displacements follows that

$$kc_g Fl\theta + c_w c_g Fl\theta - 6M_p\theta = 0$$

and the ultimate load for the combined mechanism is

$$F_u = \frac{6M_p}{(c_w + k)c_g l}. \quad (3)$$

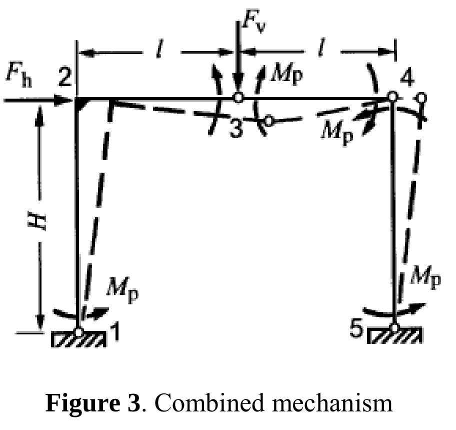

Figure 3. Combined mechanism

Equilibrium method

By using internal forces in cross section 3 and external loads, the expressions for moments M_i (i = 1,2,4,5) can be expressed as follows

$$M_1 = M_3 - Ql + kNl - kc_w c_g Fl,$$

$$M_2 = M_3 - Ql,$$

$$M_4 = M_3 + Ql - c_w c_g Fl, \quad (4)$$

$$M_5 = M_3 + Ql + kNl - c_w c_g Fl.$$

By eliminating Q and N in equations (4) the following expressions can be written:

$$2M_3 - M_2 - M_4 = c_w c_g Fl, \quad (5)$$

$$M_5 - 2M_3 - 2M_2 - M_1 = -(c_w - k)c_g Fl \quad (6)$$

Transforming expressions (5) and (6) and eliminating M_2 or M_3, we obtain

$$M_5 + M_2 - M_1 - M_4 = kc_g Fl, \quad (7)$$

$$M_5 + 2M_3 - 2M_4 - M_1 = (c_w + k)c_g Fl. \quad (8)$$

In the case of the beam mechanism the bending moments are

$$M_2 = -M_p; M_3 = M_p; M_4 = -M_p.$$

On the basis of formula (5) and the foregoing values of moments, the expression for the beam-type mechanism follows. According to the panel-type mechanism (lateral displacement) bending moments in cross sections 1, 2, 4, and 5 are

$$M_1 = -M_p; M_2 = M_p; M_4 = -M_p; M_5 = -M_p.$$

By using equation (7) and moments M_1, M_2, M_3, and M_5 we can get the expression (2).

Assuming the combined mechanism (Fig. 3) ultimate moments form in cross sections 1, 3, 4, and 5

$$M_1 = -M_p; M_3 = M_p; M_4 = -M_p; M_5 = -M_p.$$

Substituting values of moments in equation (8) we can get formula (3) for the ultimate load in the case of the combined mechanism.

FEM Analysis

For the purpose of analysis the frame is divided into linear finite elements and joints. A joint is defined as the junction of two elements. The joints have been placed along the horizontal and vertical members of the frame. The geometry of the coordinates of the joints is relative to a set of coordinate axes which are referred to as structure axes. An element is uniquely defined by specifying its number and the numbers of the joints to which it is connected. The structural characteristics of the elements are described in terms of moment of inertia, cross-sectional area, and modulus of elasticity.

NUMERICAL RESULTS AND DISCUSSION

A separate investigation is performed for each possible mechanism caused by any variation in loading. Each solution is based upon the fact that the maximum bending moment is limited to M_p i.e. plastic hinge. This permits a redistribution of other moments until the next hinge is formed. The process continues until a sufficient number of hinges have been formed to create a mechanism.

The analysis is performed on the basis of a frame with the span of $2l$ = 24 m and a different height H. Uniform rolled cross-section along the frame is assumed ($W30$). Steel with a yield limit f_y = 235 MPa is specified. The plastic moment based upon the actual frame cross-section and steel properties is M_p = 99.94 kNm. In Fig. 4 the variation of ultimate vertical load with ratio c_w at different frame geometry is shown. It is determined that for a high frame and large lateral load the load carrying capacities are established by the combined mechanism but in the case of a small lateral load by the beam-type mechanism.

By using FEM analysis it is determined that at k = 1 in order to form the plastic moment M_p the least load is in the case of the combined mechanism (F^c). It is shown in Fig. 5 that after reaching of the plastic moments in three cross-sections of frame M_i^c (i = 3, 4, 5), the redistribution of moments performs until a fourth hinge in the cross section 1 occurs (M_1^c). In the case of the beam-type mechanism the ultimate load (F^b) is higher and the values of the moments M_i^b (i = 1, 2, 3) are very close to each other. However, some redistribution of the moments is performed until the collapse of the frame takes place.

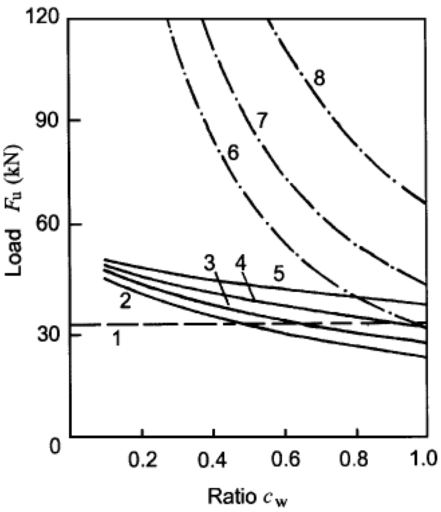

Figure 4. Ultimate load analysis at c_g =1: 1 − beam-type mechanism; combined mechanism: k = 1(2), 0.75(3), 0.5(4), 0.25(5); panel-type mechanism: k = 1(6), 0.75(7), 0.5(8)

Depending on the ratio of H to $2l$ either the mid span plastic hinge, or the two corner plastic hinges, will occur first. For small values of height to span ratio the positive moment plastic hinge forms first in the middle of span and the structure remains stable until the corner plastic hinges form. If, however, the height/span ratio is large, the corner plastic hinges occur first and the structure has reached its collapse condition only when the third hinge forms. Depending on the choice of the load combination and frame geometry the plastic design method yields slightly higher economical results. The savings of steel depending on particular case can reach 4-10%.

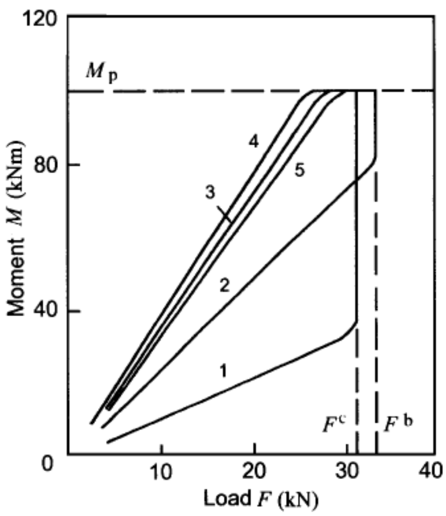

Figure 5. Mechanism type FEM analysis at k = 1 and c_g = 1; M_1^c (1), M_2^b = M_4^b (2), M_3^b = M_3^c (3), M_4^c (4), M_5^c (5)

A member subjected to a plastic moment M_p could also sustain a small axial load. However, as the axial load increases, the plastic moment has to be reduced and the design of the frame performed according to Eurocode (EN 1993-1-1, 2005). Note that it is assumed that for profiles used in the analysis, their stability limit states (lateral-torsional buckling, local flange buckling and local web buckling) are controlled.

CONCLUSIONS

The investigation shows that the ultimate load of the base fixed rigid portal frame depends on the frame geometry and the combination of gravity and wind load. It is determined that for high frame and large lateral load, the load carrying capacity is established by the combined mechanism but in the case of the small lateral load by the beam-type mechanism. Depending on the height to span ratio either the mid span plastic hinge, or the two corner plastic hinges, form first.

By using FEM analysis it is determined that for the frame with $k = 1$ and $c_w = 0.6$ the load carrying capacity is established by the combined mechanism because after the formation of the plastic moments in three cross sections the redistribution of moments takes place. The structure remains stable until the base plastic hinge occurs that results in steel savings of 4-10%.

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