PECULIARITIES OF HEAT FLOW OF INSULATION CONSTRUCTIONS IN BUILDINGS WITH COLD UNDERGROUND
SILTUMA PLŪSMU ĪPATNĪBAS ĒKU AR AUKSTO PAGRĪDI NOROBEŽOJOŠĀS KONSTRUKCIJĀS

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ABSTRACT

A popular solution in construction is an unheated garage or storehouse premises on the first or the socle floor of a building. Monolith concrete is the most common material for the first or the socle floor constructions in such cases. All surfaces of the construction disposed to outside air are insulated, and seemingly no cold bridges are developed. But the cold bridge can appear at places where the heat flow is not homogeneous. In this construction it is a place where the floor covering collides with the external wall. The thermal coefficient of linear thermal bridge has been calculated analytically with approximation and using the computer program ANSYS. Comparing both solutions, it can be stated that approximation of such types in analytical calculations does not show significant mistakes, and the obtained results are precise both qualitatively and quantitatively. The authors have offered a solution to prevent the heat loss caused by cold bridge, and it has been calculated using the ANSYS program.

Key words: cold bridge, insulation constructions of buildings, thermal coefficient

INTRODUCTION

The construction (see Fig.1.) consists of a monolith concrete wall and a covering with FIBO block wall above. All surfaces disposed to outside air are insulated.
The technical parameters used in calculation of insulated construction are presented in Tab.1.

MATERIALS AND METHODS

Without going into the root of heat flow physics, there might be an impression that there are no relevant thermal cold bridges in the presented insulated construction, because all surfaces disposed to outside air are insulated. But it is not quite true – it has to be considered that the heat flow quantity is influenced not only by thermal conductivity of the material, but also by squares through which heat flows run. In case of non homogeneous flows the latter condition may have a conclusive meaning, and it is so at the presented insulation construction. The heat flows from the covering Q_3 and FIBO blocks Q_2 through surface CB (0,2m) (Fig.1.) go into the underground wall, where the outflow conditional square consists of 2H=6m. Along with this, the thermal resistance decreases accordingly, and thermal cold bridge develops in the square ABCD.
The thermal coefficient of the cold bridge can be calculated using analytical and numeral methods. Each method has its strengths and weaknesses. The advantage of the analytical method is its relatively low calculation costs (only the specialist's wage), a possibility to use unlimited numbers and variations of physical parameters (thicknesses, conductivities, etc.) and quick calculation results.
The weakness of this method is the fact that analytical methods cannot be used for complicated constructions. Numeral methods can be used for any constructions, but calculations have to be supported by complicated and expensive computer programs. Other weaknesses - long calculation time, difficulties to vary geometrical parameters of the constructions, because each change of geometrical parameters requires redefining parameters of mathematical grid.
In order to calculate the thermal coefficient of the presented linear thermal cold bridge, we will use both analytical and numeral methods. The results from both methods will be compared.
The described insulated construction cannot be directly calculated using analytical methods. It can be done by approximation, conditionally dividing the construction into four parts:

1. Underground wall
2. Covering
3. FIBO wall
4. Cold bridge (square ABCD Fig.1.).

We will formulate mathematical physics problems for the first three objects, but for the whole cold bridge volume it will be assumed that its temperature is constant T_1.

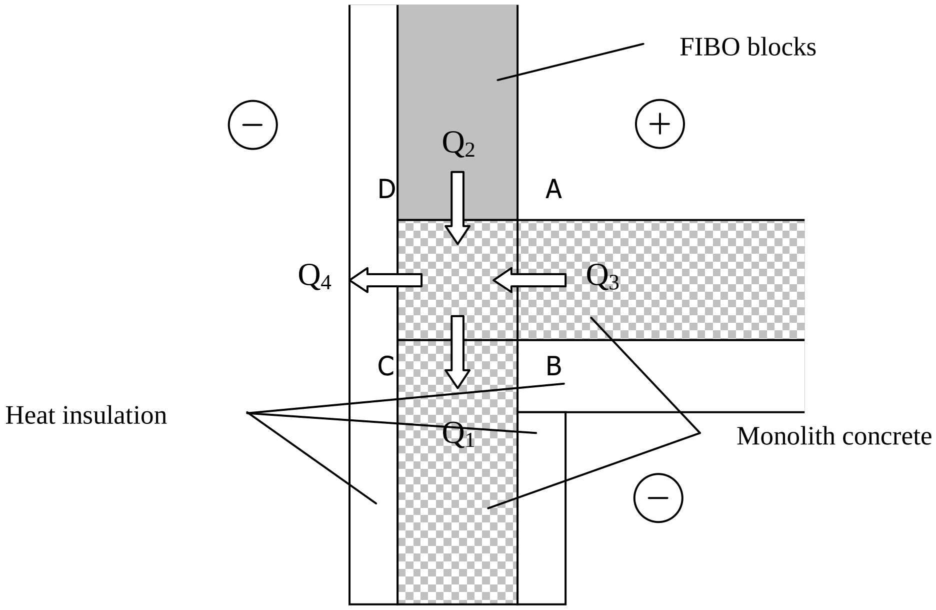

Figure 1. Insulation construction.

Table 1

Physical parameters of construction

No	Name	Thickness d, m	Thermal conductivity λ, W/(m·K)	Water steam resistance factor μ	Surface thermal resistance R_{si} or R_{se}' m^2K/W
1.	Monolith concrete	0,2	2,0	100	
2.	FIBO blocks	0,2	0,24	6	
3.	Outside wall heat insulation (mineral cotton)	0,1	0,04	1	
4.	Heat insulation under the covering (mineral cotton)	0,15	0,04	1	
5.	Foam polystyrene (used to weaken the influence of cold bridge)	0,05	0,04	60	
6.	Outside wall surface thermal resistance R_{se}				0,04
7.	FIBO blocks wall surface thermal resistance R_{si}				0,13
8.	Covering surface thermal resistance R_{si}				0,17
9.	Calculation elements: underground wall, FIBO wall height, lengths of covering assumed H=3 m.				

This temperature T_1 is not known, and the heat balance equation will be used to calculate it at the end:

$$Q_2 + Q_3 = Q_1 + Q_4 , \qquad (1)$$

where Q_2 ,Q_3 – cold bridge heat inflow from FIBO wall and monolith concrete covering, W/m.
Q_1, Q_4 – cold bridge heat outflow through underground and outside walls, W/m.

1. Calculation of underground wall temperature and heat flow

We will formulate mathematical physics problems, consisting of the heat conductivity (Laplass) equation in order to calculate the heat flow in the underground wall.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 , \qquad (2)$$

where T – temperature ^{0}C,
x, y – coordinates (Fig.2.), m.

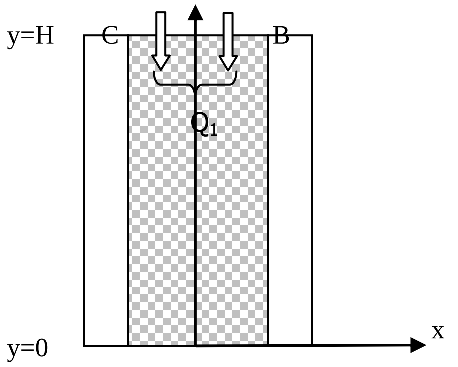

Figure 2. Underground wall calculation scheme.

The border conditions have to be defined. From the above analysis, it is known that on the line CB (Fig.1., 2.) the temperature is constant T_1, the 1th kind of border conditions is developed:

$$T\big|_{y=H} = T_1 . \qquad (3)$$

At coordinate y = 0 the underground wall collides with the ground, thus it can be assumed that the underground wall will have a strength border condition:

$$T\big|_{y=0} = Tz , \qquad (4)$$

where T_z – earth temperature, ^{0}C (in calculations assumed -18 °C).

If the border conditions (3) and (4) are stands, the solution is symmetrical in relation to y axis, and it is possible to find the solution only for the positive y values (solution is symmetrical for the negative coordinates) y>0, but on the border x = 0

$$\left.\frac{\partial T}{\partial x}\right|_{x=0} = 0 \qquad (5)$$

will stand.

On the border x =d/2, where d is the ground wall thickness, the monolith concrete construction collides with the heat insulation layer. In order to simplify the calculations (temperature division in the heat isolation layer will not be calculated – it is not relevant) we will include the heat isolation layer into the surface thermal return coefficient, at the corresponding 3rd kind border condition on the line x = d/2

$$-\lambda \left.\frac{\partial T}{\partial x}\right|_{x=d/2} = \alpha_1\left(T\big|_{x=d/2} - T_0\right), \qquad (6)$$

where λ- heat conductivity, W/(m·K);

$\alpha_1 = \dfrac{1}{R_{se} + d_i / \lambda_i}$, where-$R_{se}$=0,04 m^2 K/W;

d_i – thickness of the heat insulation layer, m;

λ_i –thermal conductivity of heat insulation, W/(m·K);

T_0 – open air temperature (in calculations assumed -20 °C).

Solving the mathematical physics problem (2 -6) with the variables separation method

$$T(x, y) = T_z + (T_1 - T_z)\frac{y}{H} + \sum_{k=1}^{\infty} A_k \mathrm{ch}\mu_k x \sin\mu_k y , \quad (7)$$

where $\mu_k = \dfrac{\pi k}{H}$;

$$A_k = -\frac{2}{\pi k} \cdot \frac{(-1)^k (T_1 - T_z) + (1-\cos\pi k)(T_z - T_0)}{\lambda\mu_k / \alpha_1 \cdot \mathrm{sh}\mu_k d/2 + \mathrm{ch}\mu_k d/2}$$

.

The heat flow Q_1 through the surface CB (Fig.1.) can be found

$$Q_1 = 2\int_0^{d/2} \lambda \left.\frac{\partial T}{\partial y}\right|_{y=H} dx = \lambda\frac{(T_1 - T_z)d}{H} + 2\lambda\sum_{k=1}^{\infty}(-1)^k A_k \mathrm{sh}\mu_k \frac{d}{2} . \quad (8)$$

2. Calculation of FIBO wall, covering temperature and heat flow

The same expressions can be used for FIBO walls and the covering temperature distribution calculations. Thus, we will show it for the covering. (Fig. 3.)

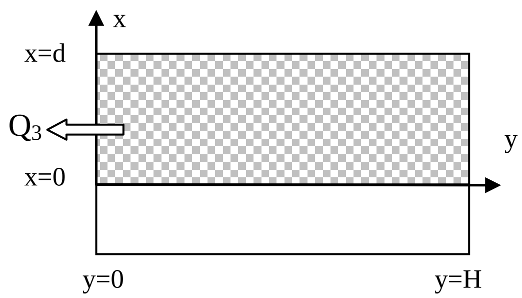

Figure 3. Covering calculation scheme

The mathematical physics problem will be made by equation (2) and border conditions:

$$T\big|_{y=0} = T_1 , \qquad (9)$$

$$\left.\frac{\partial T}{\partial y}\right|_{y=H} = 0 , \qquad (10)$$

It is assumed, that enough far from the cold bridge (y = H =3 m) the heat flow is homogenous. On the covering upper (x = d) and bottom (x = 0) surfaces, there will be the 3rd kind of border conditions:

$$\lambda \left.\frac{\partial T}{\partial x}\right|_{x=d} = \alpha_2 (T_i - T\big|_{x=d}) , \qquad (11)$$

where T_i – inside temperature (in calculations assumed 20 °C);

$\alpha_2 = \alpha_i = 1/ R_{si} = 1/ 0{,}17$, W/(m^2·K);

$$\lambda \frac{\partial T}{\partial x}\bigg|_{x=0} = \alpha_1 (T|_{x=0} - T_0)\ , \qquad (12)$$

where in the coefficient α_1, the heat insulation layer under the covering is included

$$\alpha_1 = \frac{1}{R_{se} + d_i / \lambda_i}\ .$$

Solving the mathematical physics problem (2, 9 – 12) with the variables separation method

$$T(x,y) = T_1 + \sum_{k=0}^{\infty} (C_k \mathrm{sh}\mu_k x + D_k \mathrm{ch}\mu_k x) \sin\mu_k y\ , \quad (13)$$

where $\mu_k = \frac{\pi(k+0{,}5)}{H}$, k=0, 1, 2, ...

$$C_k = \frac{2}{\pi(k+0{,}5)}\ \mathrm{x}$$

$$\mathrm{x}\, \frac{(T_1 - T_0)(\lambda\mu_k/\alpha_2 \cdot \mathrm{sh}\mu_k d + \mathrm{ch}\mu_k d) + (T_i - T_1)}{(1 + \lambda^2 \mu^2{}_k /(\alpha_1\alpha_2)) \cdot \mathrm{sh}\mu_k d + \lambda\mu_k (1/\alpha_1 + 1/\alpha_2) \mathrm{ch}\mu_k d}$$

$$D_k = \frac{2}{\pi(k+0{,}5)}\ \mathrm{x}$$

$$\mathrm{x}\, \frac{(T_i - T_0)\lambda\mu_k/\alpha_1 - (T_1 - T_0)(\lambda\mu_k/\alpha_2 \cdot \mathrm{ch}\mu_k d + \mathrm{sh}\mu_k d)}{(1 + \lambda^2 \mu^2{}_k /(\alpha_1\alpha_2)) \cdot \mathrm{sh}\mu_k d + \lambda\mu_k (1/\alpha_1 + 1/\alpha_2) \mathrm{ch}\mu_k d}$$

Accordingly - Q_2 and Q_3 will be found in the form

$$Q_{2;3} = -\int_0^d \lambda \frac{\partial T}{\partial y}\bigg|_{y=0} dx = -\lambda \sum_{k=0}^{\infty} C_k(\mathrm{ch}\mu_k d - 1) + D_k \mathrm{sh}\mu_k d\ . (14)$$

In order to find the heat flow Q_4 through the outside wall, homogenous solution formulas will be used

$$Q_4 = \frac{T_1 - T_0}{R_{se} + d_i/\lambda_i}\ . \qquad (15)$$

Including the found heat flows (8, 14, 15) in expression (1), we get transcendent equation in relation to unknown cold bridge temperature T_1. This equation can be easily solved numerically, using Microsoft Excel tool Solver.

The heat flow through the covering and FIBO wall is not homogenous, especially close to the place where they collide. These can be calculated using temperature distribution (13)

$$Q_{gr,\,FIBO} = \int_0^H \lambda \frac{\partial T}{\partial x}\bigg|_{x=d} dy = \lambda \sum_{k=0}^{\infty} C_k \mathrm{ch}\mu_k d + D_k \mathrm{sh}\mu_k d\ . \quad (16)$$

If the heat flow is homogenous, the analogical value of expression (16) can be calculated using the formula

$$Q_{hom} = \frac{(T_i - T_0)H}{R_{si} + R_{se} + d/\lambda + d_i/\lambda_i}\ . \qquad (17)$$

In expressions (16, 17) it needs to be considered, that the covering and FIBO wall calculations have different R_{si} and insulation layer thickness d_i. Then the linear cold bridge thermal coefficient Ψ W/(m·K), which is calculated on 1 K temperature difference between the outside and inside air, can be found as the following:

$$\Psi_{gr;FIBO} = \frac{\lambda}{T_i - T_0} \sum_{k=0}^{\infty} C_k \mathrm{ch}\mu_k d + D_k \mathrm{sh}\mu_k d - \frac{H}{R_{si} + R_{se} + d/\lambda + d_i/\lambda_i}\ , \qquad (18)$$

But the thermal coefficient of the common linear cold bridge:

$$\Psi = \Psi_{gr} + \Psi_{FIBO}\ . \qquad (19)$$

In order to assess the analytical solution approximation (formula 1 – cold bridge has constant temperature T_1) accurately, this task was solved numerically using the program ANSYS.

RESULTS AND DISCUSSION

The influence of the cold bridge in the building construction expresses as significant decrease of temperature of the FIBO wall and covering (Fig.1.) close to the point A. This temperature distribution can be calculated by expression (13), and it is presented in Fig. 4., 5. The curves 1 and 2 accordingly – temperature on inner and outer surfaces where they collide with the heat insulation layer (Fig.3. x=d and x=0).

As it is seen from the calculations, Fig.1 point A forecasted temperature is 7,7 °C, which on the FIBO wall reaches the balanced temperature 18,5 °C approx. 30 cm distance, but on monolith concrete covering 18,3 °C 1 m distance. Thus, the micro climate of the premises is significantly worsened. Fig.4. curve 2 – the existing minimum does not have physical basis. That is reached by approximations used in calculations, that all cold bridges have the same temperature.

The temperature distribution in the underground wall on y axis (Fig.2.) is shown in Fig.6. It can be seen that temperature distribution is not linear in nature. At y=0 $T=T_z=-18$ °C, but at the wall upper part y=H=3 m T=7,7 °C.

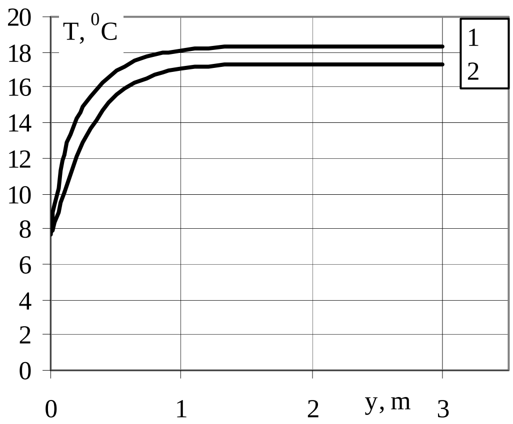

Figure 4. Temperature distribution in FIBO wall. 1 – on the inner surface of the wall; 2 – on the outer surface under heat insulation

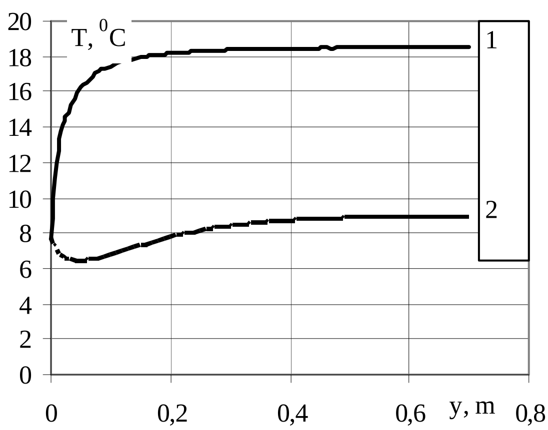

Figure 5. Temperature distribution in covering. 1 – on the inner surface of the covering; 2 – on the outer surface of the covering above heat

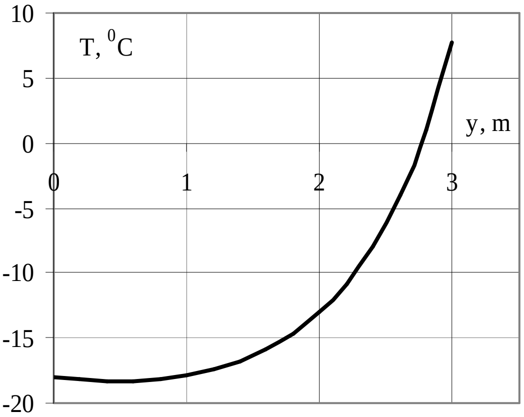

Figure 6. Temperature distribution on underground wall axis line.

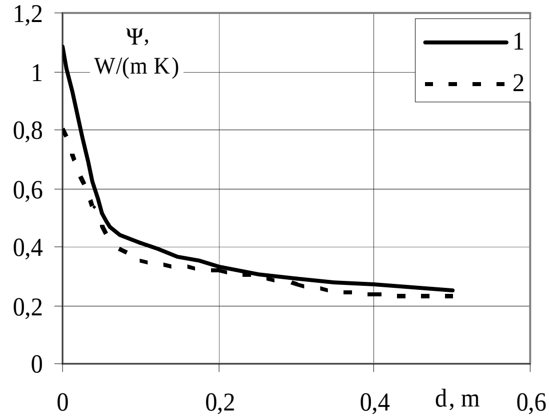

Figure 7. Linear cold bridge thermal coefficient Ψ depending on underground wall heat insulation thickness. Curve 1 has been obtained analytically, 2– numerically by program ANSYS.

The Building Regulation LBN 002-01 defines the normative Ψ_R and maximal values Ψ_{RM} for the linear cold bridge thermal coefficient Ψ. For example, for dwelling-houses at the temperature factor k=1 the values are the following: Ψ_R=0,2W/(m K); Ψ_{RM}=0,25W/(m K). When using Tab.1. physical values, the thermal coefficient of the linear cold bridge of dwelling-houses, depending on the underground wall heat insulation thickness, is presented in Fig.7.

As it is seen, the thermal coefficient of the cold bridge increases the normative value even at heat insulation layer thickness of 0,5 m. Thus, the increase of the heat insulation layer of underground does not avert significantly the influence of cold bridge and it cannot be suggested as problem solution. At the same time when comparing the analytical and numerical solutions, it is seen that approximation of the analytical methods on average gives 10% higher value of the thermal coefficient.

Another possibility is to increase the heat insulation thickness under the monolith concrete covering. Fig.8. shows the cold bridge thermal coefficient dependence on the thickness of the heat insulation layer of the covering.

From the graph it is visible that when increasing the heat insulation layer thickness under monolith concrete covering, the thermal coefficient of cold bridge even increases. It is understandable from physical considerations, because heat flow goes into the direction of lower thermal resistance, and the flow goes through the cold bridge when the resistance of the covering becomes stronger. Thus, additional heat insulation under monolith concrete covering will not solve the problem but even intensify it. The problem solution is a heat insulation layer of 15 cm under the monolith concrete covering, which (Fig.1.) collides with the point B, dividing into two parts.

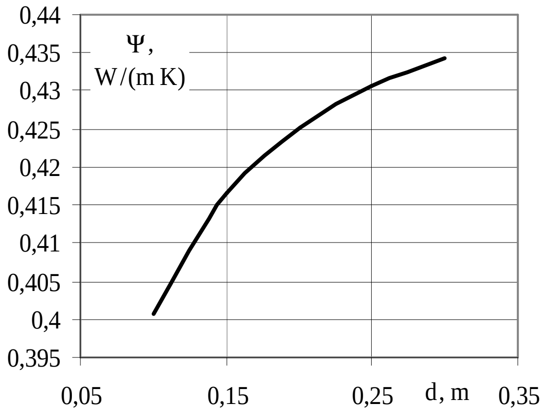

Figure 8. Linear cold bridge thermal coefficient Ψ depending on thickness of heat insulation under monolith concrete covering.

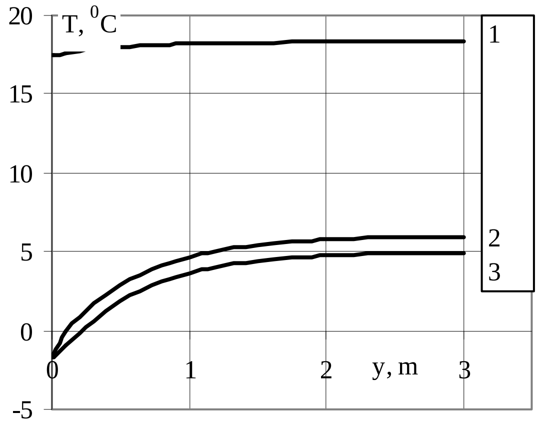

Figure 9. Temperature distribution in covering after cold bridge heat insulation.

The first part of 10 cm thickness is the mineral cotton under the monolith concrete covering, but the second part of 5 cm thickness of foam polystyrene has to be placed inside the premise above the covering. The thermal coefficient of the covering U=0,246 W/(m^2·K) will not be changed, but the thermal coefficient of cold bridge will be decreased to Ψ =0,237 W/(m K). The value is calculated analytically, and it is expected that the real Ψ value will be even 10% less, what coincides with the LBN 002-01 requirements.
The temperature distribution depending on the thickness of the heat insulation layer on y axis (analogical to Fig.5.) is presented in Fig. 9. The trend line No.1 is temperature inside the premises above the foam polystyrene heat insulation layer, the trend line No.2 – under it, at the level where foam polystyrene and monolith concrete collide, the trend line No.3 – under monolith concrete, at the level where it collides with the mineral cotton heat insulation layer.
As it is seen from the curve 1, the problems raised by the cold bridge in the premises have been eliminated.

Additionally, the risk of condensate development has to be assessed. The calculation shows that at outside temperature above -10 °C, the condensate risk does not exist. If the outside temperature is under -10 °C for a long time, little condensate can develop on the surface where monolith concrete and polystyrene collide. The condensate dries up during the drying period. The condensate risk possibility is eliminated if a steam barrier above the polystyrene layer is used in the premise.

CONCLUSIONS

1. A cold bridge develops in the analysed construction, in spite of heat insulation of all surfaces which are disposed to outside air.
2. The cold bridge thermal coefficient can be decreased to the Building Regulation LBN 002-01 requirements, by dividing the covering heat insulation layer into two parts:
   - Mineral cotton layer of 10 cm under the covering;
   - Polystyrene layer of 5 cm above the covering.

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