II MATERIALS AND STRUCTURES

TOPOLOGY OPTIMIZATION OF MULTI-LAYERED COMPOSITE STRUCTURES

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ABSTRACT

Numerical analysis is performed in order to investigate deformation behavior and strength of antisymmetric laminates under tension caused by axial load. It is determined that antisymmetric orientation of external layers of in-plane balanced laminate can be used to ensure the necessary adaptive warping and strength of composite under action of tensile stresses. The efficiency of the use of carbon and glass epoxy laminates as well as wooden laminate (birch plywood) in wind rotor blades is analysed.

Key words: Laminates, Compliance, Stiffness, Force Stress, Couple Stress, Curvature, Strength

INTRODUCTION

The use of multilayered laminates with unidirectional fiber reinforced plies is well established for lightweight constructions and special applications (Hirano and Todoroki, 2005; Pagano and Soni, 1988)). Because of their high strength and stiffness characteristics, coupled with low weight, composite materials are more attractive for engineering applications than conventional isotropic materials. In addition, because of the highly anisotropic properties of single plies, composite materials allow tailoring of the laminate behavior to the structural needs.

In multilayered fibre-reinforced composites with variable fibre orientations, the directional expansion of the unidirectional single layers due to thermal effects, moisture absorption and chemical shrinkage leads to a discontinuous residual stress field over the laminate thickness. In the case of unsymmetrical laminate plates, these residual stresses can cause different multistable out-of-plane deformations (Brauns and Rocens, 1994; Diaconu and Sekine, 2003).

This study focuses on the purposeful adaptation of the residual stresses dependent on the stacking sequence of layered system in order to realise either laminates with defined multistable deformation states or to design adaptive structures. For the adjustment of laminate

curvatures to technical requirements, methods are developed that can efficiently be applied to find an optimal laminate lay-up dependent on the material properties and loading conditions (Hansel and Becker, 2000). Furthermore, failure analysis is carried out in order to assess the resulting residual stress state with respect to first-ply failure.

In general, laminates can be designed to provide the desired strength and stiffness characteristics required for specific applications. The material anisotropy can be exploited to induce coupling between deformation modes. The use of fibre reinforced composite rotor blades enables a number of possible passive aerodynamic control options. By using adaptive wind rotor blades with twist coupling there is a possibility to keep good, steady power production and smooth out unwanted peaks in loading. The blades may be made wholly or partially from carbon fibre, which is a lighter, but costlier material with high strength. A numerical analysis is performed to investigate deformation behavior and strength of in-plane balanced antisymmetric laminates under tension caused by axial centrifugal load.

STRESS AND STRAIN RELATIONSHIPS OF LAYERED STRUCTURE

Composite materials can be constructed by bonding together several structural elements to form an integral structure. The properties and orientation of each element have to be chosen such that the composite is able to meet the design requirements of strength and stiffness. It is also necessary to know the behavior of the material under various environmental conditions, such as exposure to water, low and high temperature.

Within a layered composite of thickness *h*, deformation of one layer is constrained by the other ones of different orientations, and hence stresses arise in each layer. In general case, the stresses in the elementary layers are different and the stress state of the composite is inhomogeneous. By using a static equivalent system of average force stresses

σ_j and moment stresses μ_j acting on a unite volume of the composite material, the constitutive relations for the mid-plane strains ε_i^0 and the curvatures κ_i in matrix notations are given by

$$\left[\frac{\boldsymbol{\varepsilon}^0}{\boldsymbol{\kappa}}\right] = \left[\frac{\boldsymbol{\alpha} \;|\; \boldsymbol{\beta}}{\boldsymbol{\beta}^{\mathrm{T}} \;|\; \boldsymbol{\delta}}\right] \cdot \left[\frac{\boldsymbol{\sigma}}{\boldsymbol{\mu}}\right], \quad (1)$$

where $\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\delta}$ are compliances of layered composite; superscript T denotes transposition operation.
The force stresses and moment stresses in layered composite are calculated by averaging

$$\sigma_j = \int_{-h/2}^{h/2} \tilde{\sigma}_j^{(k)} dx_3 \,, \quad (2)$$

$$\mu_j = \int_{-h/2}^{h/2} \tilde{\sigma}_j^{(k)} x_3 dx_3 \,. \quad (3)$$

The stresses $\tilde{\sigma}_j^{(k)}$ in the *k*th elementary layer in the coordinate of composite $\{x_i\}$ (i = 1, 2, 3) can be determined by using the layer stiffness A'_{ij} in the local coordinate system $\{x'_i\}$ and stress transformation matrix (Tsai and Hahn 1980).
In the general case, the compliance matrices in (1) are represented in terms of composite stiffness:

$$\boldsymbol{\alpha} = \mathbf{S} + \mathbf{SBCBS}\,; \quad (4)$$

$$\boldsymbol{\beta} = -\mathbf{SBC}\,; \quad (5)$$

$$\boldsymbol{\delta} = \mathbf{C} \quad (6)$$

with

$$\mathbf{C} = [\mathbf{D} - \mathbf{BSB}]^{-1}. \quad (7)$$

The stresses in the elementary layers (Fig. 1) in the coordinates of composite $\{x_i\}$ can be determined by using the layer stiffness A'_{ij} in the local coordinate system $\{x'_i\}$ and stress transformation matrix. The stiffness components in (4)–(7) are evaluated by integrations:

$$[A_{ij}, B_{ij}, D_{ij}] = \int_{-h/2}^{h/2} \tilde{A}_{ij}^{(k)} [1, x_3, x_3^2]\, dx_3 \,. \quad (8)$$

In (8) the stiffness matrix $[\tilde{A}_{ij}]^{(k)}$ of the elementary layer in the coordinate system $\{x_i\}$ can be determined by using the transformation formula. The compliance matrix in (4), (5) and (7) is $[S_{ij}] = [A_{ij}]^{-1}$.

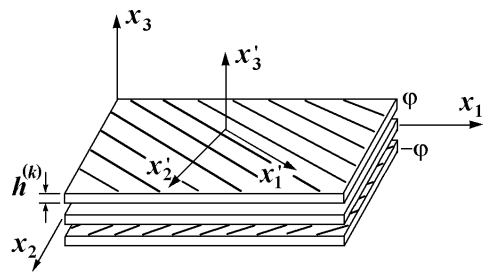

Figure 1. Multilayer model of antisymmetric laminated composite structure.

STRENGTH ANALYSIS OF DESIGNED LAMINATE

Predicting the failure of structural components is usually accomplished by comparing the stresses to the material strength limits. A number of failure criteria have been proposed, however, the main issue is whether there is any interaction between the modes of failure. Experimental observations for fiber-reinforced materials show interactions between the failure modes. For example, shear failure is expected to occur more easily if, in addition to the shear stress, there is also a normal tensile stress. The most frequently used failure criteria taking account of this effect are polynomial criteria advised by Malmeister (Malmeister, 1966) and Tsai and Wu (Tsai and Wu, 1971). This criterion is easy to apply because it does not distinguish between different failure modes. On the other hand, it takes into account the interaction between the in-plane stresses in different directions.

A more general form of the failure criterion for orthotropic materials, i.e., materials with two mutually perpendicular planes of symmetry in mechanical properties, under plane stress state is expressed as

$$\begin{aligned} f(\sigma_{ij}) &= F_{11}\sigma_{11} + F_{22}\sigma_{22} + 2F_{12}\sigma_{12} + \\ &+ F_{1111}\sigma_{11}^2 + +F_{2222}\sigma_{22}^2 + 4F_{1212}\sigma_{12}^2 + \\ &+ 2F_{1122}\sigma_{11}\sigma_{22} + +4F_{1112}\sigma_{11}\sigma_{12} + \\ &+ 4F_{2212}\sigma_{22}\sigma_{12} = 1. \end{aligned} \quad (9)$$

The coefficients in the stress function (9) are the components of tensors F_{ij} and F_{ijkl}. They are determined by means of the strength tensor $R_{\alpha\beta\gamma}$, where α = 0, 11, $\overline{11}$; β = 0, 22, $\overline{22}$; γ = 0, 12, $\overline{12}$. The index 0 denotes that the given stress component is absent; the bar over the index is employed to indicate a compressive component. Using the strength values obtained experimentally, the coefficients are represented in the following form:

$$F_{11} = \frac{R_{\overline{1}100} - R_{1100}}{R_{1100}R_{\overline{1}100}};\ F_{22} = \frac{R_{0\overline{2}20} - R_{0220}}{R_{0220}R_{0\overline{2}20}};$$
$$F_{1111} = \frac{1}{R_{1100}R_{\overline{1}100}};\ F_{2222} = \frac{1}{R_{0220}R_{0\overline{2}20}}; \qquad (10)$$
$$2F_{1122} = \frac{F_{11} - F_{22}}{R_{11\overline{2}20}} + F_{1111} + F_{2222} - \frac{1}{R^2_{11\overline{2}20}};$$
$$4F_{1212} = \frac{1}{R_{0012}R_{001\overline{2}}}.$$

The axes 1 and 2 of the strength envelope (ellipsoid) are set in the plane defined by σ_{11}~σ_{22} and axis 3 is parallel to the axis of shear stresses $\sigma_{12.}$ The components of the tensor of the strength surface, F_{11} and F_{22}, express the displacement of the centre of the ellipsoid along the axes 1 and 2, respectively. The angle of rotation of the ellipsoid relative to axis 1 is a function of the component F_{1122}.
The mentioned criterion defines an envelope in stress space: if the stress state lies outside of this envelope, then failure is predicted. The failure mechanism is not specifically identified, although inspection of the relative magnitudes of the terms in (9) gives an indication of the likely contribution of the modes.

RESULTS AND ANALYSIS

Considering laminates with fixed total thickness, the objective function could be the effective elastic or strain characteristic, while constraints are imposed on other properties. The layer orientation design problem involves design of a laminate with a single orientation angle and the laminate can also be more complex, providing additional layers with fixed orientations. In practice, fibre-reinforced composite laminates for conventional stiff rotor blades incorporate a combination of unidirectional plies to support radial loads and provide sufficient bending stiffness, and 45° plies to restrict shear and torsion. Representing the pertinent characteristic as a function of undetermined face layer orientation angle ±φ and determining the optimum can be solved as optimisation problem.
Graphical procedure can be used for the design of laminates with prescribed in-plane properties (Miki, 1983). The procedure is suitable for in-plane balanced angle-ply laminates made up of stacks of layers with different orientation angles ±φ. In addition to the balanced angle-ply groups, unidirectional layers with principal material axis aligned with the axes of the laminate, i.e. 0° and 90°, can be included in the stacking sequence.
For a in-plane balanced composite laminates elastic characteristics of the laminate can be determined by using lamination parameters that contain all the relevant information associated with the stacking sequence. The lamination parameters can be determined in the following way

$$V_{1\{\mathbf{A,B,D}\}} = \int_{-h/2}^{h/2} \cos 2\varphi \{1, x_3, x_3^2\} dx_3; \qquad (11)$$

$$V_{2\{\mathbf{A,B,D}\}} = \int_{-h/2}^{h/2} \sin 2\varphi \{1, x_3, x_3^2\} dx_3; \qquad (12)$$

$$V_{3\{\mathbf{A,B,D}\}} = \int_{-h/2}^{h/2} \cos 4\varphi \{1, x_3, x_3^2\} dx_3; \qquad (13)$$

$$V_{4\{\mathbf{A,B,D}\}} = \int_{-h/2}^{h/2} \sin 4\varphi \{1, x_3, x_3^2\} dx_3. \qquad (14)$$

In general case, lamination parameters and material invariants (Tsai and Pagano, 1968) can be used for determination of the mentioned above stiffness characteristics A_{ij}, B_{ij} and D_{ij}. Two parameters $V_{1\mathbf{A}}$ and $V_{3\mathbf{A}}$ after normalizing were used to determine optimum design.
By using the lamination parameter diagram (Fig. 2), it is possible to determine the region of allowable combinations of lamination parameters. For a laminate of total thickness h, where the volume fraction of layers with $\pm\varphi_i$ orientation angles is v_i, normalized lamination parameters are given as

$$\overline{V}_1 = \frac{V_{1\mathbf{A}}}{h} = \sum_{i=1}^{N} v_i \cos 2\varphi_i; \qquad (15)$$

$$\overline{V}_3 = \frac{V_{3\mathbf{A}}}{h} = \sum_{i=1}^{N} v_i \cos 4\varphi_i, \qquad (16)$$

where N is the number of different ±φ groups.
The values of the lamination parameters are always bounded, i.e., $-1 \le (\overline{V}_1, \overline{V}_3) \le 1$. For a laminate with one orientation angle the two parameters are related as

$$\overline{V}_3 = 2\overline{V}_1^2 - 1. \qquad (17)$$

Values of all possible combinations of the lamination parameters are, therefore, located along the boundary line ABC (Fig. 2). The points A, B and C correspond to laminates with 0°, ±45, and 90° orientation angles, respectively. Any point inside the boundary line corresponds to laminates with two or more fiber orientation.
Composite laminates for rotor blades incorporate a combination of unidirectional plies to support radial centrifugal loads and provide sufficient bending stiffness, and 45° plies to restrict shear and torsion. Using graphical procedure and representing the needed characteristic as a function of undetermined

external layer orientation angle φ the optimisation problem is solved. The procedure is suitable for in-plane balanced angle-ply laminates made up of stacks of layers with different orientation angles ±φ. The analysed laminates consist of 9 layers that form an in-plane balanced antisymmetric system with stacking sequence [φ, 45, –45, 90, 0, 90, –45, 45, –φ]. In the lamination parameter diagram (Fig. 2) the point L corresponds to the orientation of external layers φ = 0° but the point N to φ = 90°. The point M belongs to the laminate configuration with φ = ±25° when laminates indicate the maximum warping under action of axial load.

In Figures 3, 4 and 5 the curvature of the laminates determined by using relationship (1) and strength function determined according to (9) is shown. The warping of wooden laminate is greater in comparison with carbon epoxy and glass epoxy laminate, but in all cases there are allowable intervals, where the curvature changes from zero until maximum, but the stress level is allowable, i.e., $f(\sigma_{ij}) < 1$.

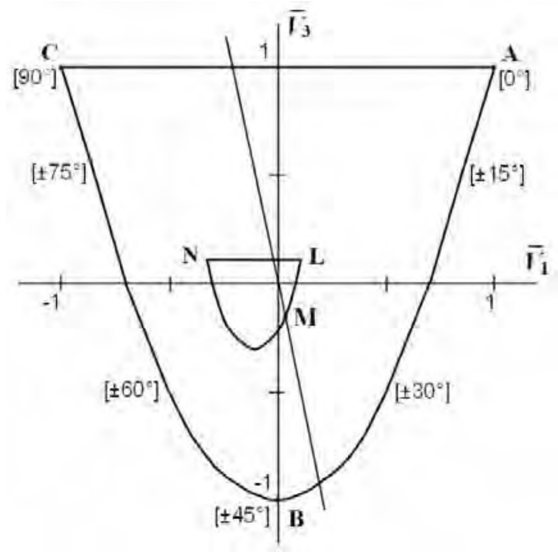

Figure 2. Design area for in-plane lamination parameters: ABC – defined stacks of layers; LMN – analysed laminates.

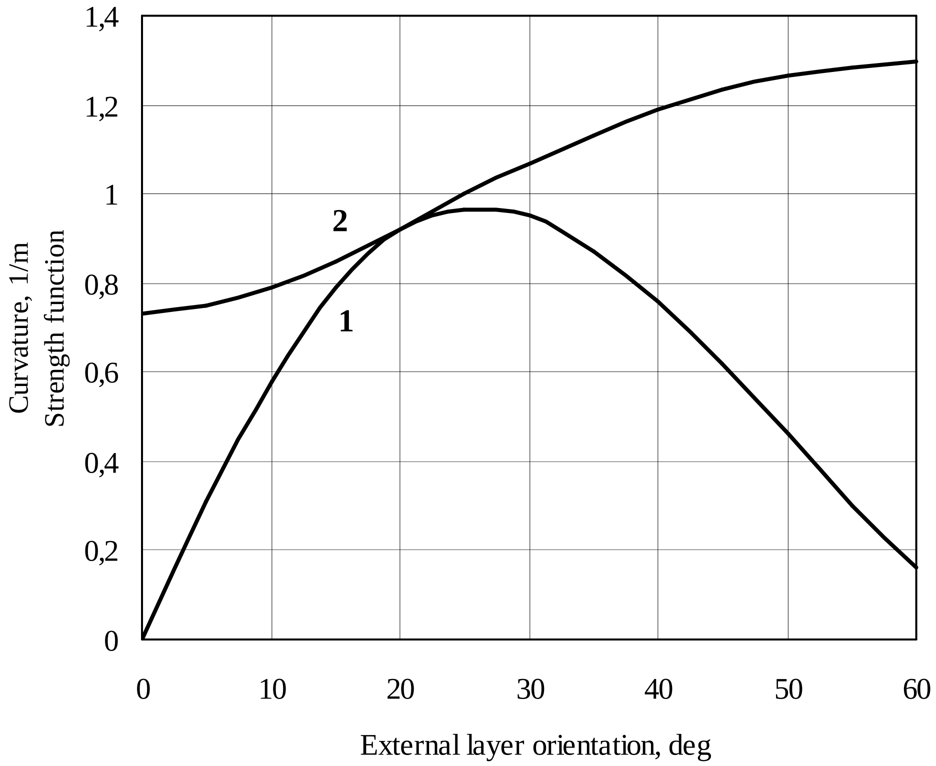

Figure 3. Curvature (1) and strength function (2) vs. external layer orientation. for carbon epoxy laminate.

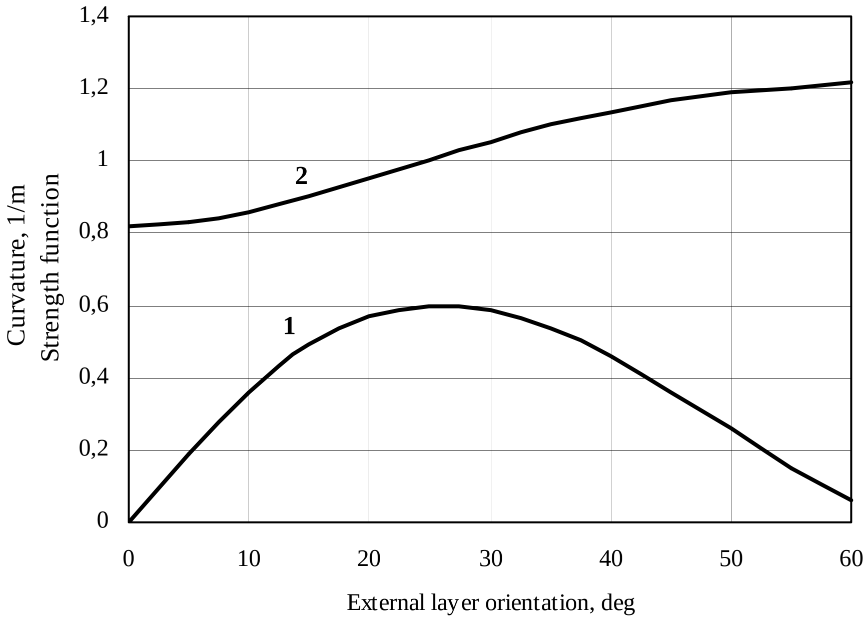

Figure 4. Curvature (1) and strength function (2) vs. external layer orientation for E-glass epoxy laminate.

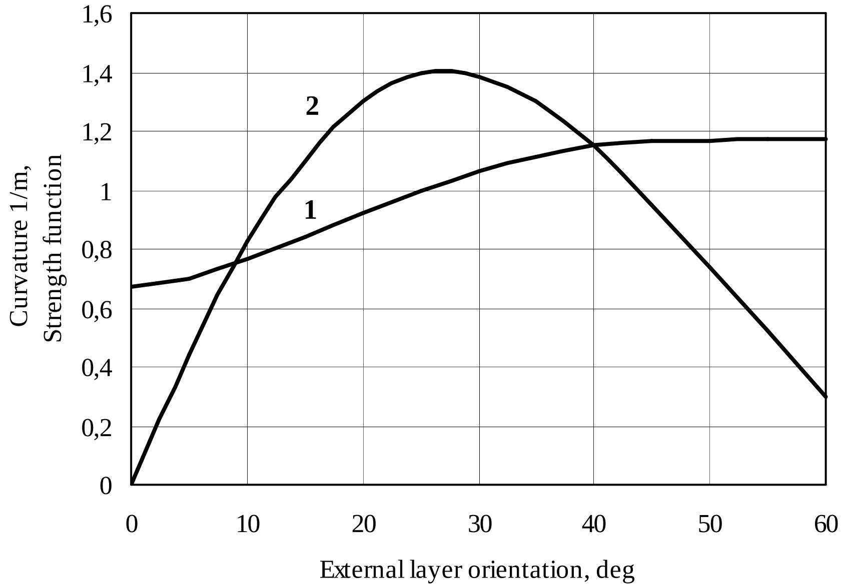

Figure 5. Curvature (1) and strength function (2) vs. external layer orientation for birch plywood.

Table 1

Strength and economical characteristics of laminates

Laminate	Carbon/epoxy	Glass/epoxy	Plywood
Average stresses at ultimate, MPa	270	80	45
Price, USD/m^2	46 700	22 000	33

CONCLUSIONS

Antisymmetric orientation of external layers of in-plane balanced laminate can be used to ensure the necessary adaptive warping and strength of the laminate under action of axial load.
The use of stretching-twisting coupling can be applied to provide a control mechanism of rotor blades which does not have any parts moving relative to each other, and which is therefore maintenance-free.
For in - plane balanced laminates with anti-symmetric orientation of external layers under action of axial force adaptive warping and strength of the laminate can be ensured.
For the analysed laminates the allowable interval for external layer orientation is 0°–25°·Maximum twisting can be obtained in case of birch plywood but the price of this laminate is relatively low.

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