THEORY AND PRACTICE IMPROVING OF MATHEMATICAL PROCESSING OF GEODETIC MEASUREMENTS FOR LAND MANAGEMENT AND CADASTRE WORKS

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Abstract
The article generalizes theoretical and practical addition to the components of mathematical processing of geodetic measurements, which are the basis and used to support work in land management and cadastre. The research identifies general components of mathematical processing of geodetic measurements and components which are primarily used in the field of land management and cadastre. Completed investigation of the theoretical and practical foundations of these components and developed theoretical and practical additions improve the results of mathematical processing of geodetic measurements.

Introduction

The main means of obtaining credible and relevant information for land management and cadastre are surveying. Traditional mathematical processing of the results from geodetic measurements and methodology are not suitable for processing of the results for the needs of land management and cadastre. Existing scientific and applied problem in land management and cadastre in Ukraine is the necessity of actual and reliable information; there is incompleteness of the theoretical and practical foundations of mathematical processing of geodetic measurements which could be used and have an impact on the work of land management.
The research works devoted to the study of problems of mathematical processing of geodetic measurements of national and foreign scientists include: Bolʹshakov V.D., Viduyev M.H., Voytenkj S.P., Gauss K.F., Zazulyak P.M., Idelʹson M.I., Linnyk YU.V., Karpinsʹkyy YU.O., Kondra H.S., Mazmishvili A.I., Markuze YU.I., Mohylʹnyy S.H., Smirnov M.V., Tretyak K.R., Shulʹts R.V. etc.
The authors conducted the research on: establishing values and dependencies between parameters likelihood function and the condition of maximum probability of occurrence of random errors together, developing proposals [1, 11 - 13]; establishing patterns and dependencies between the averages, their systematization [2, 5 - 8]; definition and refinement of dependency properties of random errors, development of criteria and formulas for obtaining consideration of rounding errors [3, 4]; analysis of different types of double measurements, criteria for a systematic error, developing methods and obtaining formulas for excluding of systematic error, justification for rejection of correlation coefficient [9, 10].

Research methodology and materials

Ordinary Least Squares (OLS) is essential for adjustment of geodetic measurements. The study mainly used:
• condition of maximum probability of random errors together

$$P_{\Delta_1,\Delta_2,\ldots,\Delta_n}=\left(\frac{1}{\sqrt{2\pi}}\right)^n e^{-\frac{1}{2}\left(\frac{\Delta_1^2}{m_1^2}+\frac{\Delta_2^2}{m_2^2}+\ldots+\frac{\Delta_n^2}{m_n^2}\right)}\cdot\frac{d\Delta_1}{m_1}\cdot\frac{d\Delta_2}{m_2}\cdot\ldots\cdot\frac{d\Delta_n}{m_n} \qquad (1)$$

where the highest value of probability of sets of true errors $P_{\Delta_1,\Delta_2,\ldots,\Delta_n}$ will be provided if the exponent is minimal, that expression in parentheses (1)

$$\sum_{i=1}^{n}\frac{\Delta_i^2}{m_i^2}=\min \text{ or } \sum_{i=1}^{n}\frac{v_i^2}{m_i^2}=\min \qquad (2)$$

(2)

- R. Fisher’s method of maximum likelihood estimation (MLE) with function

$$L = f(y_1, y_2, ..., y_n) = (2\pi)^{-\frac{n}{2}} (\sigma_0^{2n})^{-\frac{1}{2}} [\det Q]^{-\frac{1}{2}} \exp(-\frac{1}{2\sigma_0^2}[y - M_y]^T Q^{-1}[y - M_y]) \quad (3)$$

where $y_1, y_2, \dots, y_n$ – results of measurements,

σ_0^2 – dispersion of weight unit,

$Q = P^{-1}$ – matrix of inverse weights,

M_y – mathematical expectation of values y, when best estimates M_y and σ_0^2 are obtained on condition

lnL = max.

Also, MLE is used to study the simple arithmetic average and the total arithmetic average, dispersion by one measurement and dispersion of weight unit, respectively.

Discussion and results

As a result of mathematical transformations, expression (3) takes the following form to unequally accurate measurements:

$$L = (2\pi)^{-\frac{n}{2}} \frac{1}{m_{y_1} \cdot m_{y_2} \cdot ... \cdot m_{y_n}} e^{-\frac{r}{2}} \text{ or } L = (2\pi)^{-\frac{n}{2}} \frac{(p_{y_1} \cdot p_{y_2} \cdot ... \cdot p_{y_n})^{\frac{1}{2}}}{\sigma_0^n} e^{-\frac{r}{2}} \quad (4)$$

and equally accurate measurements –

$$L = (2\pi)^{-\frac{n}{2}} \frac{1}{m^n} e^{-\frac{r}{2}} = \frac{1}{(2\pi)^{\frac{n}{2}} \cdot m^n \cdot e^{\frac{r}{2}}}. \quad (5)$$

According to the research of dependency of functions arguments (1) and (3) and their varieties, it is found that the expression (2) will be equal $\sum_{i=1}^{n} \frac{\Delta_i^2}{m_i^2} = n$, $\sum_{i=1}^{n} \frac{v_i^2}{m_i^2} = r$ or $\sum_{i=1}^{n} \frac{p_i v_i^2}{\mu^2} = r$ [1, 11 – 13]. That is the exponent of value e function (1) and (3) regardless of the types of its records is always a half of all or redundant measurements with minus sign, and does not depend on the values of true errors or amendments and mean squared error (MSE) as well as on the value from which deviations are calculated. An increase of the number of redundant measurements does not cause increasing value of likelihood function, but instead - reduces its value.

Dispersion is derived from the expression (3) is $\sigma_0^2 = \frac{\Sigma p v^2}{n}$. When it comes to simple arithmetic average or total arithmetic average, the difference of value in denominator (n aбо n – 1), especially when we have large number of n will not substantially affect the value of dispersion. Instead, during adjustment of values between which there are independent functional connections, the number of excess and all measurements may be different, and the difference between them is already high. Therefore, estimation of measurement results will be displaced, and this must be considered.

The value of likelihood function (3) and its logarithm depends on three variables: number of all measurements, number of redundant measurements and value of MSE of weight unit in the case of unequally accurate measurements or value of MSE in the case of equally accurate measurements (Fig. 1) [13].

By the analysis of results of likelihood function parameters calculations and their relationships, it was found that the function and its logarithm acquire positive values only if MSE is less than 0.398942.... With the value MSE σ = 0.398942… the function depends on the number of redundant measurements, and its logarithm is equal to a half the number of redundant measurements minus sign. When increasing MSE more than 0.398942… (even when σ = 0.4), the function and its logarithm is rapidly declining as the number of redundant and all measurements (Fig. 1).

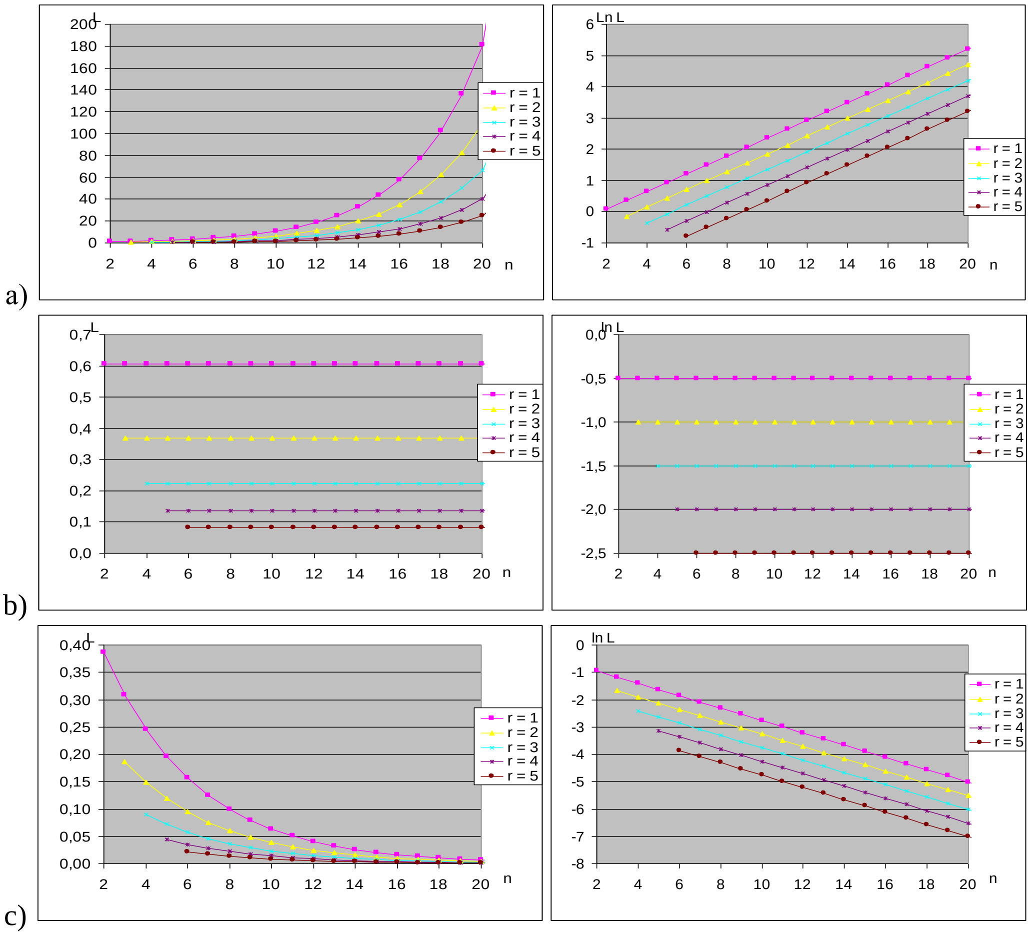

Fig. 1. Dependence of function L and ln$L$ on number of measurements n
a) when σ = 0,3; b) when σ = 0,398942 ...; c) when σ = 0,5

Similar results were obtained in the study of justifying the likelihood function simple arithmetic average and the total arithmetic average. That is to say about a maximum or minimum of this function is not entirely correct. Therefore, for justification for OLS of simple arithmetic average and the total arithmetic average it is offered not to use likelihood function (3) [11 - 13].

Since the beginning of the state land cadastre, the question about the rounding of coordinates of the vertices rotation angles of land plot and of its square is important, because in case of recalculating of land by rounded coordinates, other values of square and length of land are obtained, especially during the recalculation of coordinates from one coordinate system to another. A differential approach is proposed to solving this problem depending on the value of square, configuration, the number of vertices rotation angles, boundaries of land. Three criterions of importance of systematic error rounding of coordinates [3, 4] are obtained to determine when rounding coordinates can be neglected, and when they cannot be neglected:

$$m_{S_{окр}} \le 0{,}054\, m_S, \qquad m_{S_{окр}} \le 0{,}031\, m_S, \qquad m_{S_{окр}} \le 0{,}020\, m_S. \tag{6}$$

The resultant functional dependence MSE of area due to rounding coordinates and calculating the threshold rounding respectively:

$$m_{S_{окр}} = \frac{a}{2\sqrt{3}} \sum_{i=1}^{n} D_{(i+1)-(i-1)}, \quad a = \frac{2\sqrt{3}\, m_{S_{окр}}}{\sum_{i=1}^{n} D_{(i+1)-(i-1)}}, \tag{7}$$

where a – a limiting error of coordinates rounding, equal to a half of the last mark,
D – diagonal between vertices rotation angles $(i + 1)$ and $(i – 1)$.

Theoretical and practical study of harmonious, geometric, quadratic, power average by measuring one value [6, 7] is completed. In the case of equally accurate measurements with established properties, the average values are systematized in three groups:

- group of simple arithmetic average: $\bar{x}_a = \left(\frac{x_1{}^k + x_2{}^k + \ldots + x_n{}^k}{n} \right)^{\frac{1}{k}}$;

- group of simple harmonic average: $\bar{x}_h = \left(\frac{n}{x_1{}^{-k} + x_2{}^{-k} + \ldots + x_n{}^{-k}} \right)^{\frac{1}{k}}$;

- group of simple geometric average: $\bar{x}_g = \left(x_1^k \cdot x_2^k \cdot \ldots \cdot x_n^k \right)^{\frac{1}{nk}} = \left(x_1 \cdot x_2 \cdot \ldots \cdot x_n \right)^{\frac{1}{n}}$.

The obtained results in respect of averages in the case of unequally accurate measurements are also systematized in three groups:

- group of total arithmetic average:

$$\bar{x}_3 = (q_1 x_1^k + q_2 x_2^k + \ldots + q_n x_n^k)^{\frac{1}{k}} \text{ or } \bar{x}_3 = \left(\frac{p_1 x_1^k + p_2 x_2^k + \ldots + p_n x_n^k}{\Sigma p}\right)^{\frac{1}{k}};$$

- group of total harmonic average:

$$\bar{x}_h = (q_1 x_1^{-k} + q_2 x_2^{-k} + \ldots + q_n x_n^{-k})^{-k} \text{ or } \bar{x}_h = \left(\frac{p_1 x_1^{-k} + p_2 x_2^{-k} + \ldots + p_n x_n^{-k}}{\Sigma p} \right)^{-k};$$

- group of total geometric average:

$$\bar{x}_g = (x_1^{kq_1} \cdot x_2^{kq_2} \cdot \ldots \cdot x_n^{kq_n})^{\frac{1}{k}} = x_1^{q_1} \cdot x_2^{q_2} \cdot \ldots \cdot x_n^{q_n} \text{ or}$$

$$\bar{x}_g = (x_1^{kp_1} \cdot x_2^{kp_2} \cdot \ldots \cdot x_n^{kp_n})^{\frac{1}{k\Sigma p}} = (x_1^{p_1} \cdot x_2^{p_2} \cdot \ldots \cdot x_n^{p_n})^{\frac{1}{\Sigma p}}.$$

If the exponent will increase or decrease, differences between the values of average will also increase or decrease accordingly. The exponent does not affect the value of a simple and total geometric average [6, 7].

The investigations of averages on equally accurate measurements and unequally accurate measurements provided an opportunity to establish relationships between them [2, 5 - 8]. Also, inequality between values of investigated averages, its MSE of weight unit, weights of averages and its MSE was established. For all averages the sum of squared normalized deviations is equal to the number of redundant measurements $\Sigma t^2 = n - 1$ [6, 7].

The third property of deviations measurement from total arithmetic average was established: sum of products of weights of measurements and squared deviations measurement results of total arithmetic average is the sum of squares products of all the unique differences and appropriate measurement of weights divided by sum of weights of all measurements, i.e.

$$\sum_{i=1}^{n} p_i v_i^2 = \frac{\sum_{\substack{i=1 \\ j=2}}^{n} d_{i-j}^2 p_i p_j}{\sum_{i=1}^{n} p_i}. \quad (8)$$

Traditionally during mathematical processing of the results from geodetic measurements MSE of weight unit is calculated by the known formula:

$$\mu = \sqrt{\frac{\Sigma p_x vv}{n-1}} \ . \tag{9}$$

Thus, the expression (9) use weight of measurements, despite the fact that weights are not equal weights of deviations [5]. MSE calculation of one measurements, in the case of equally accurate measurements, was performed by well-known Bessel's formula. But weight deviation p_v, which is used in the Bessel's formula is not equal to one because $p_v = n/(n+1)$. And, if considering weight deviations, the Bessel's formula will take the following form:

$$m = \sqrt{\frac{n\Sigma v^2}{n^2-1}} \ . \tag{10}$$

It is established that to adjustment is the MSE of weight unit by measured values, and after adjustment should be used MSE of weight units by aligned values. To improve the estimation of unequally accurate measurements of one measurement, weight deviations is offered to use during calculating MSE of weight unit. In the case of equally accurate measurements offered during MSE calculating (Bessel's formula) the weight deviations are taken into account, if $n < 20$ [5].

The results of the study show that different importance criteria of a systematic error give reason to make opposite conclusions about its importance, and depends on further estimation of the accuracy of double equally accurate and unequally accurate measurements [9, 10]. It is therefore proposed to use only one criterion $|\Sigma d_i| \le 0{,}25\Sigma|d_i|$ or $|\Sigma p_d d_i| \le 0{,}25\Sigma|p_d d_i|$ (here: d_i – differences; p_d – weight differences).

Systematic errors should be considered not only for the performance estimation of accuracy, but also for excluding them from the average values of double equally accurate and unequally accurate measurements. Therefore relevant dependencies are established which are described by formulas for all cases of double measurements and their usage is investigated [9, 10].

As a result of the research Abbe's criterion established dependencies by which we can calculate the parameter q, using only the difference between equally accurate measurements. In the case of unequally accurate measurements dependencies were established by which we can also calculate the parameter q herewith using the difference between their measurements and weight. These dependencies provide an opportunity to use Abbe's criterion during processing and unequally accurate measurements and testing various hypotheses.

Conclusions and recommendations

1. Failure and incompleteness of theoretical and practical bases on certain areas of mathematical processing of geodetic measurements were established that can be used to support work in land management and cadastre.

2. Theoretical and practical additions to components of mathematical processing of geodetic measurements were developed:

- it was determined that in the case justification for OLS, simple arithmetic average and total arithmetic average not using maximum likelihood condition of random errors together and the method of maximum likelihood by R. Fisher;
- criteria significance of rounding coordinate errors was established and the formulas for pre-calculating rounding precision coordinates when calculating the land area were developed;
- in accordance with specified properties averages by the results of equally accurate and unequally accurate measurements were systematized in three groups, respectively, dependency of their systematization of groups and relationship was established, averages by their MSE and weight (except simple arithmetic average and total arithmetic average) were determined, the third property of deviations from total arithmetic average was established;
- dependencies were obtained and the method was developed with using Abbe's criterion for determining systematic error and verification of various hypotheses using only differences between equally accurate and unequally accurate measurements;
- usage of only one criterion importance of systematic error in the case of equally accurate and unequally accurate double measurements, respectively was proved and proposed. The developed

formulas of exclusion systematic error for all cases of double-measurements and importance of systematic error determined not only by differences but also with average values of double-measurements, which reduce the residuals.

3. The obtained theoretical and practical research results improved the efficiency and optimized the mathematical processing of geodetic measurements not only in the field of cadastre and land management, but also in any direction of using geodetic measurements.

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